Geometry of Hohfeldian Relations

7 September 2021, 36th CILC Conference @ Parma (online)

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Contributions of this work

- Enriching the analysis of Hohfeld’s framework of normative concepts
- Individuating and formalizing three distinct families of power:
    - change-centered, force-centered, outcome-centered
- Formulating Aristotelian squares of opposition for all Hohfeldian relationships, thus clarifying the logical relevance of Hohfeld’s analysis
- Expanding squares into hexagons to solve symmetry issues that arise in the literature
- Highlighting connections between the various notions of power, and between potestative and deontic concepts

Hohfeld’s analysis

Hohfeld is known for presenting a taxonomy of concepts used in legal/judicial reasoning, centered around two levels (see, e.g., Hohfeld 1913):

- first-order (or **deontic**) concepts, such as duty, claim, no-claim and liberty;
- second-order (or **potestative**) concepts, such as power, liability, immunity and disability.

All these concepts are read as *ternary relations* among two normative parties and a certain behaviour (here, a type of action).

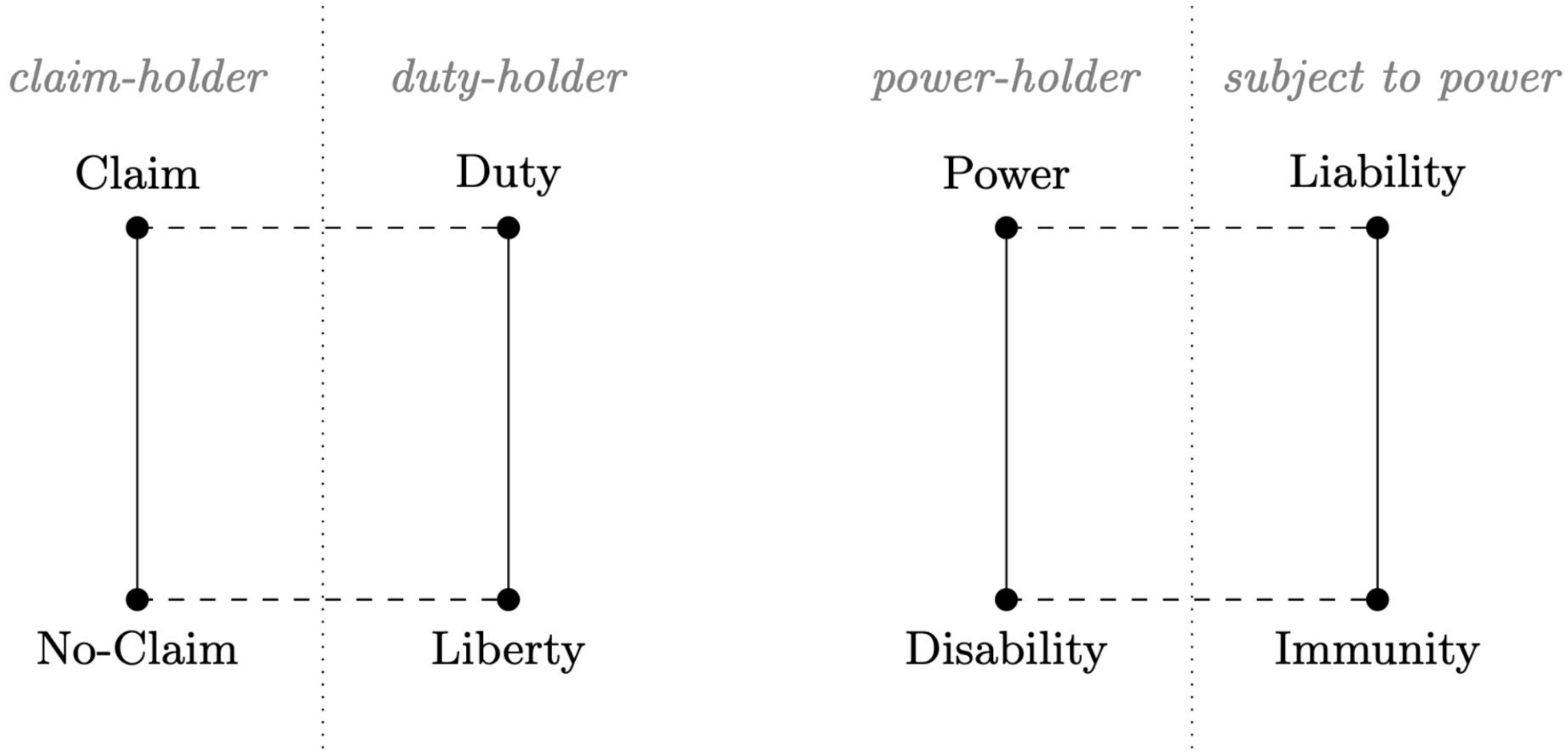
Hohfeldian vs. Aristotelian squares

Hohfeldian squares are two diagrams originally proposed by W.N. Hohfeld to represent conceptual interactions between deontic notions and between potestative notions. However, they lack a clear logical interpretation.

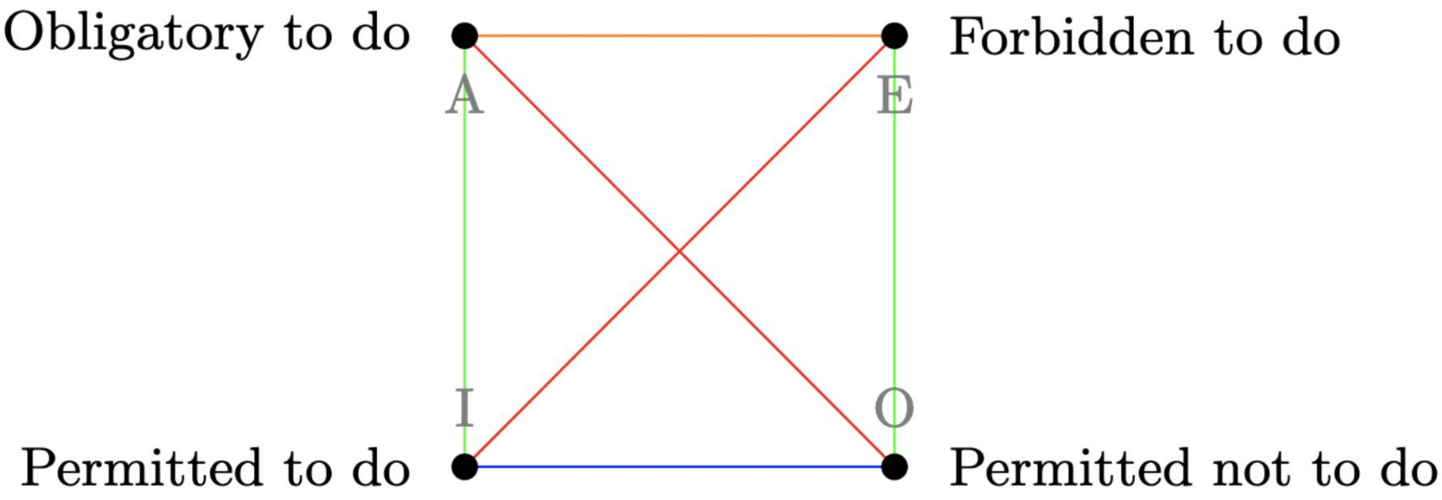
Aristotelian squares are traditional diagrams providing a geometrical representation of logical theories in terms of relations such as contradiction, contrariety, subcontrariety and subalternation. Each square works on a finite set of formulas and yields a decidable theory with respect to a logical system.

Key issue: *What is the relation between Hohfeldian squares and Aristotelian ones?*

Hohfeldian squares



Aristotelian square for basic deontic modalities



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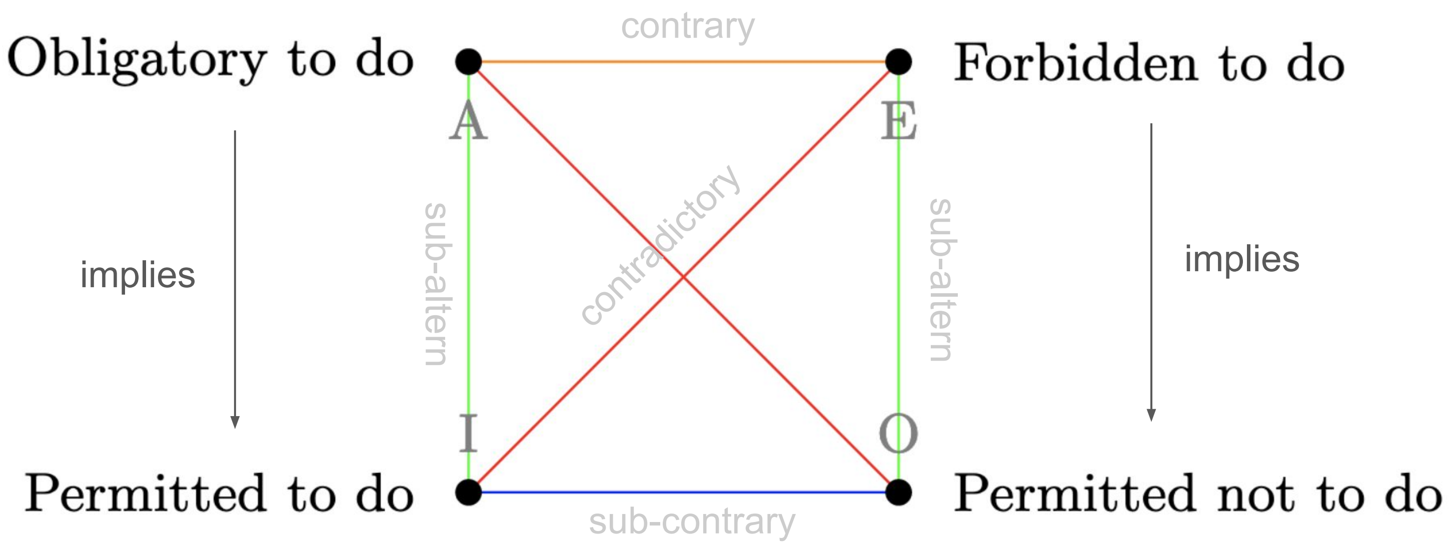


First-order Hohfeldian concepts

Ternary relations among two normative parties and an action type: claim, no-claim, duty and liberty. We formalize these in terms of a language of predicate logic with individual terms for normative parties (x,y,... are variables and p,q,... are constants) and for action-types (α,β, … are variables and A,B,... are constants). Furthermore, we use action complementation.

- Each of these can be taken as primitive and used to define the others;

$$\begin{aligned} \mathsf{NoClaim}(x, y, A) &\equiv \neg\mathsf{Claim}(x, y, A) \\ \mathsf{Duty}(y, x, A) &\equiv \mathsf{Claim}(x, y, A) \\ \mathsf{Liberty}(y, x, A) &\equiv \neg\mathsf{Claim}(x, y, \overline{A}) \end{aligned}$$

First-order Hohfeldian concepts

Ternary relations among two normative parties and an action type: claim, no-claim, duty and liberty.

- Each of these can be taken as primitive and used to define the others;
- Each choice of a primitive notion gives rise to an Aristotelian square.

$$\mathsf{NoClaim}(x, y, A) \equiv \neg\mathsf{Claim}(x, y, A)$$
$$\mathsf{Duty}(y, x, A) \equiv \mathsf{Claim}(x, y, A)$$
$$\mathsf{Liberty}(y, x, A) \equiv \neg\mathsf{Claim}(x, y, \overline{A})$$

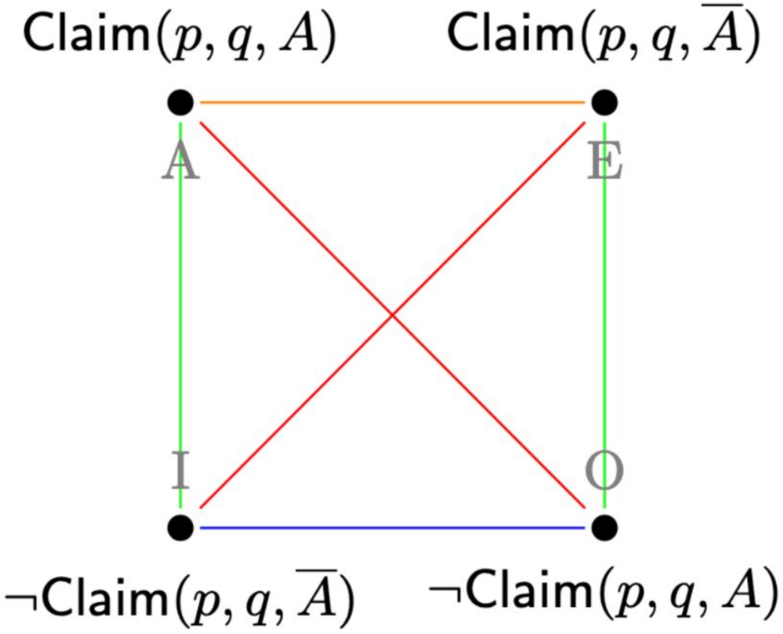
sub-alternation relation

$$\mathsf{Claim}(x, y, A) \rightarrow \neg\mathsf{Claim}(x, y, \overline{A})$$

set of 4 positions

$$\mathsf{DR} = \{\mathsf{Claim}(p, q, A), \mathsf{Claim}(p, q, \overline{A}), \neg\mathsf{Claim}(p, q, \overline{A}), \neg\mathsf{Claim}(p, q, A)\}$$

Claim-based square of opposition



Second-order Hohfeldian concepts

Ternary relations among two normative parties and an action type: power, liability, disability and immunity.

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"Canonic" form of power: the ability or competence to create a claim/duty

$$\mathsf{Power}(x, y, A) \equiv \exists \beta : \mathit{Ability}(x, \beta, \mathsf{Claim}(x, y, A))$$

agent

action-type
stimulus

configuration
manifestation

Second-order Hohfeldian concepts

Ternary relations among two normative parties and an action type: power, liability, disability and immunity.

- Each of these can be taken as primitive and used to define the others;

$$\mathsf{Disability}(x, y, A) \equiv \neg\mathsf{Power}(x, y, A)$$

$$\mathsf{Liability}(y, x, A) \equiv \mathsf{Power}(x, y, A)$$

$$\mathsf{Immunity}(y, x, A) \equiv \neg\mathsf{Power}(x, y, A)$$

Second-order Hohfeldian concepts

Ternary relations among two normative parties and an action type: power, liability, disability and immunity.

- Each of these can be taken as primitive and used to define the others;
- However, we treated power as defined in terms of ability;
- *Does a notion of power defined in terms of ability give rise to an Aristotelian square?*

$$\mathsf{Disability}(x, y, A) \equiv \neg\mathsf{Power}(x, y, A)$$

$$\mathsf{Liability}(y, x, A) \equiv \mathsf{Power}(x, y, A)$$

$$\mathsf{Immunity}(y, x, A) \equiv \neg\mathsf{Power}(x, y, A)$$

we individuate three distinct forms of power and build the corresponding Aristotelian squares.

Change-centered power

Originally analysed by Sumner (1987) and, more rigorously, by O’Reilly (1995).

The notion of power at its core concerns the ability of a normative party *p* to affect another normative party *q* with respect to a certain relation *R*. We redefined it using *ability*...

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Focusing on "canonic" power, R is about a duty

$$\begin{aligned}\mathsf{Power}_{\mathsf{OReilly}}(x, y, B, A) \equiv\ & \mathit{Ability}(x, B, \mathsf{Claim}(x, y, A)) \\ \vee\ & \mathit{Ability}(x, B, \mathsf{Claim}(x, y, \overline{A})) \\ \vee\ & \mathit{Ability}(x, B, \neg\mathsf{Claim}(x, y, A))\end{aligned}$$

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able to create a duty upon y

able to create a prohibition upon y

able to create a (partial) liberty upon y

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$$\mathsf{Power}^{+}(x, y, A) \equiv \exists\beta : \mathsf{Power}_{\mathsf{OReilly}}(x, y, \beta, A) \longleftarrow$$

The agent can do something changing R

$$\mathsf{Power}^{-}(x, y, A) \equiv \exists\beta : \neg\mathsf{Power}_{\mathsf{OReilly}}(x, y, \beta, A) \longleftarrow$$

The agent can do something without changing R

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\end{aligned}
$$

sub-alternation relation

$$\mathsf{Power}^{+}(x, y, A) \equiv \exists \beta : \mathsf{Power}_{\mathsf{OReilly}}(x, y, \beta, A)$$

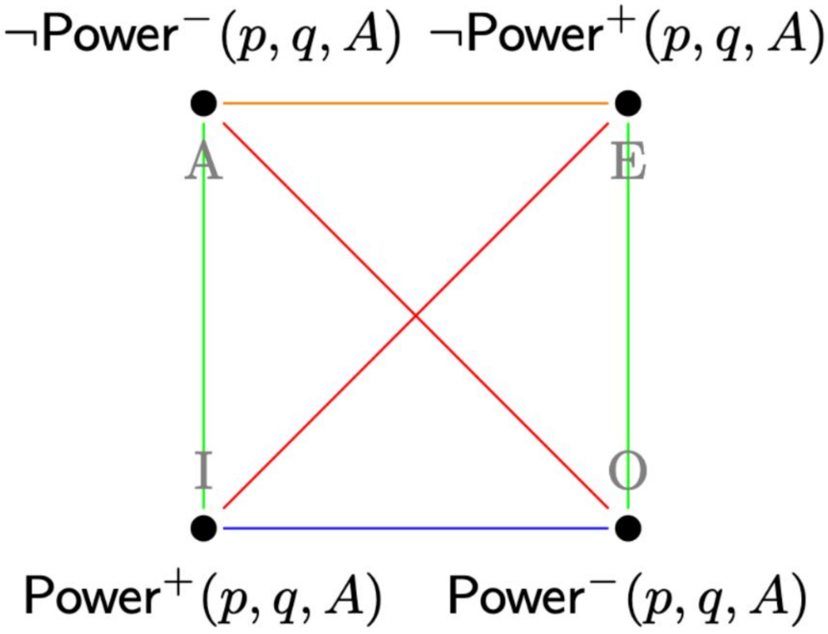
$$\neg\mathsf{Power}^{-}(x, y, A) \rightarrow \mathsf{Power}^{+}(x, y, A)$$

$$\mathsf{Power}^{-}(x, y, A) \equiv \exists \beta : \neg\mathsf{Power}_{\mathsf{OReilly}}(x, y, \beta, A)$$

set of 4 positions

$$\mathsf{PR}^{\pm} = \{\mathsf{Power}^{+}(p, q, A), \mathsf{Power}^{-}(p, q, A), \neg\mathsf{Power}^{-}(p, q, A), \neg\mathsf{Power}^{+}(p, q, A)\}$$

Change-centered power: square of opposition



Force-centered power

First observed in Sileno et al. (2015): the notion of power can be put in analogy to physical notions as *attraction* and *repulsion* towards a certain relation.

- attraction corresponds to **positive-force power** (the power to attract [*create a duty to perform*] a certain action type A).
- repulsion corresponds to **negative-force power** (the power to repel [*create a prohibition to perform*] a certain action type A).

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same stimulus

opposite manifestations

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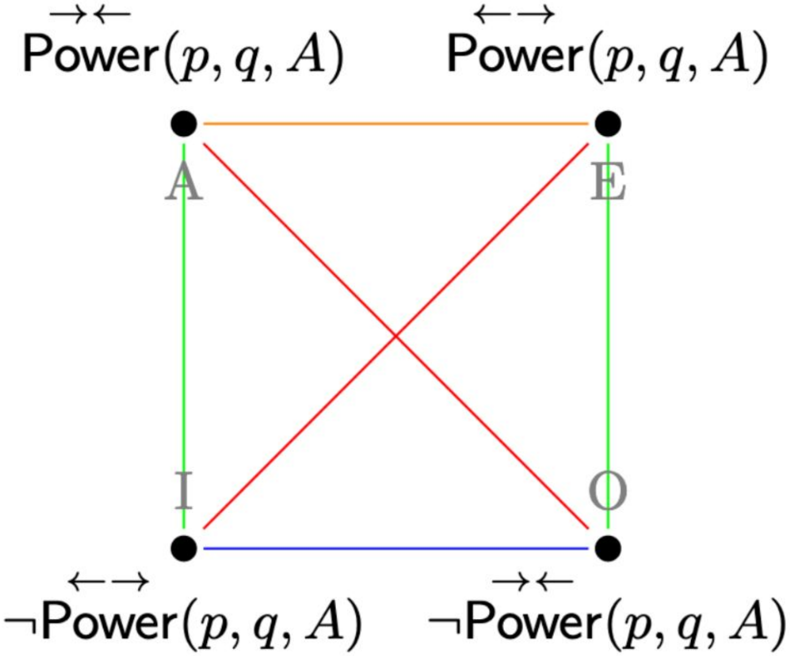
sub-alternation relation

$$\overset{\rightarrow\leftarrow}{\mathsf{Power}}(x, y, A) \rightarrow \neg\overset{\leftarrow\rightarrow}{\mathsf{Power}}(x, y, A)$$

set of 4 positions

$$\mathsf{PR}^{\leftrightarrows} = \{\overset{\rightarrow\leftarrow}{\mathsf{Power}}(p, q, A), \overset{\leftarrow\rightarrow}{\mathsf{Power}}(p, q, A), \neg\overset{\leftarrow\rightarrow}{\mathsf{Power}}(p, q, A), \neg\overset{\rightarrow\leftarrow}{\mathsf{Power}}(p, q, A)\}$$

Force-centered power: square of opposition



Outcome-centered power

The notion of power at its core is centered around the outcome produced.

We can distinguish between the **power to issue** a duty (canonic power) and the **power to release** from a duty.

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$$\overline{\mathsf{Power}}(x, y, A) \equiv \exists \beta : \mathit{Ability}(x, \beta, \neg\mathsf{Claim}(x, y, A))$$

sub-alternation relation

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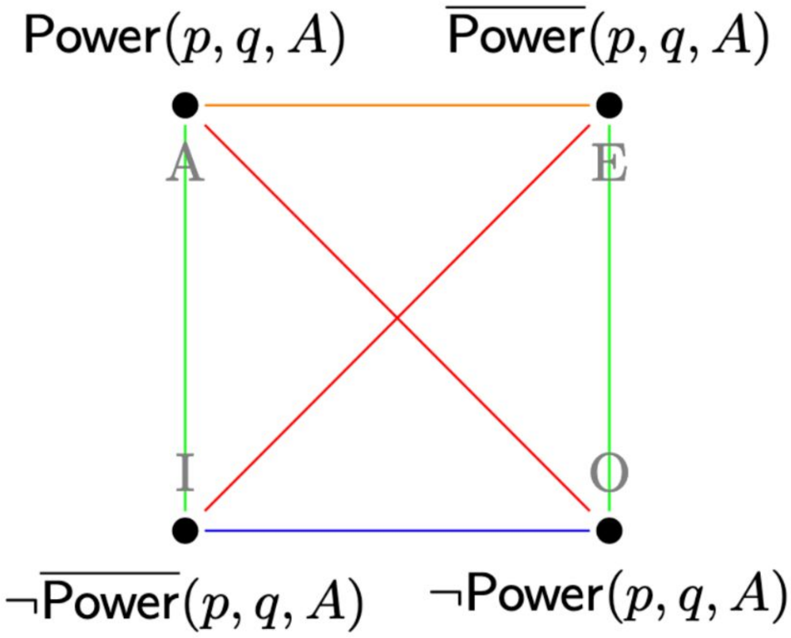
set of 4 positions

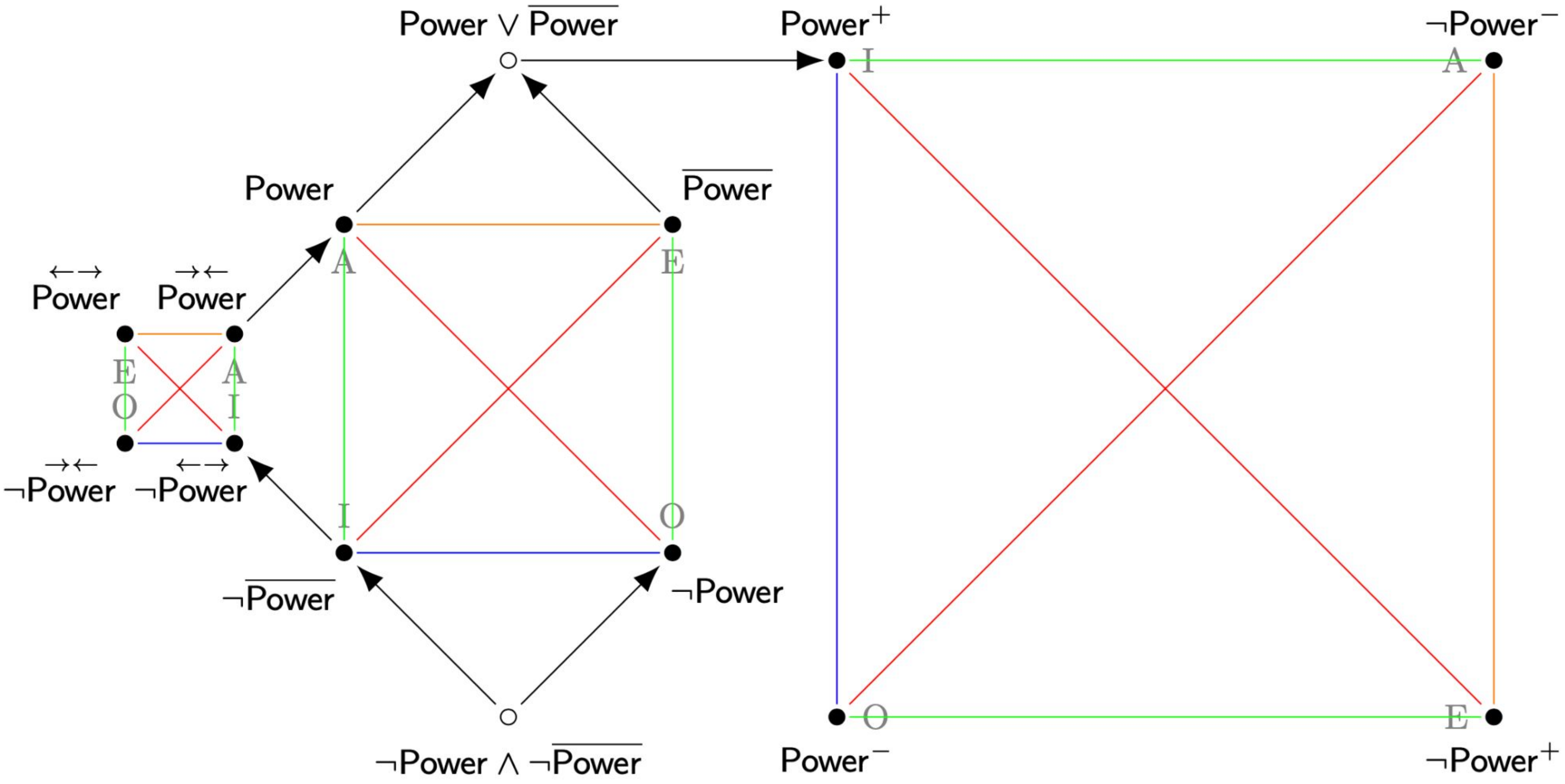
$$\mathsf{PR} = \{\mathsf{Power}(p, q, A), \overline{\mathsf{Power}}(p, q, A), \neg\overline{\mathsf{Power}}(p, q, A), \neg\mathsf{Power}(p, q, A)\}$$

Outcome-centered power: square of opposition

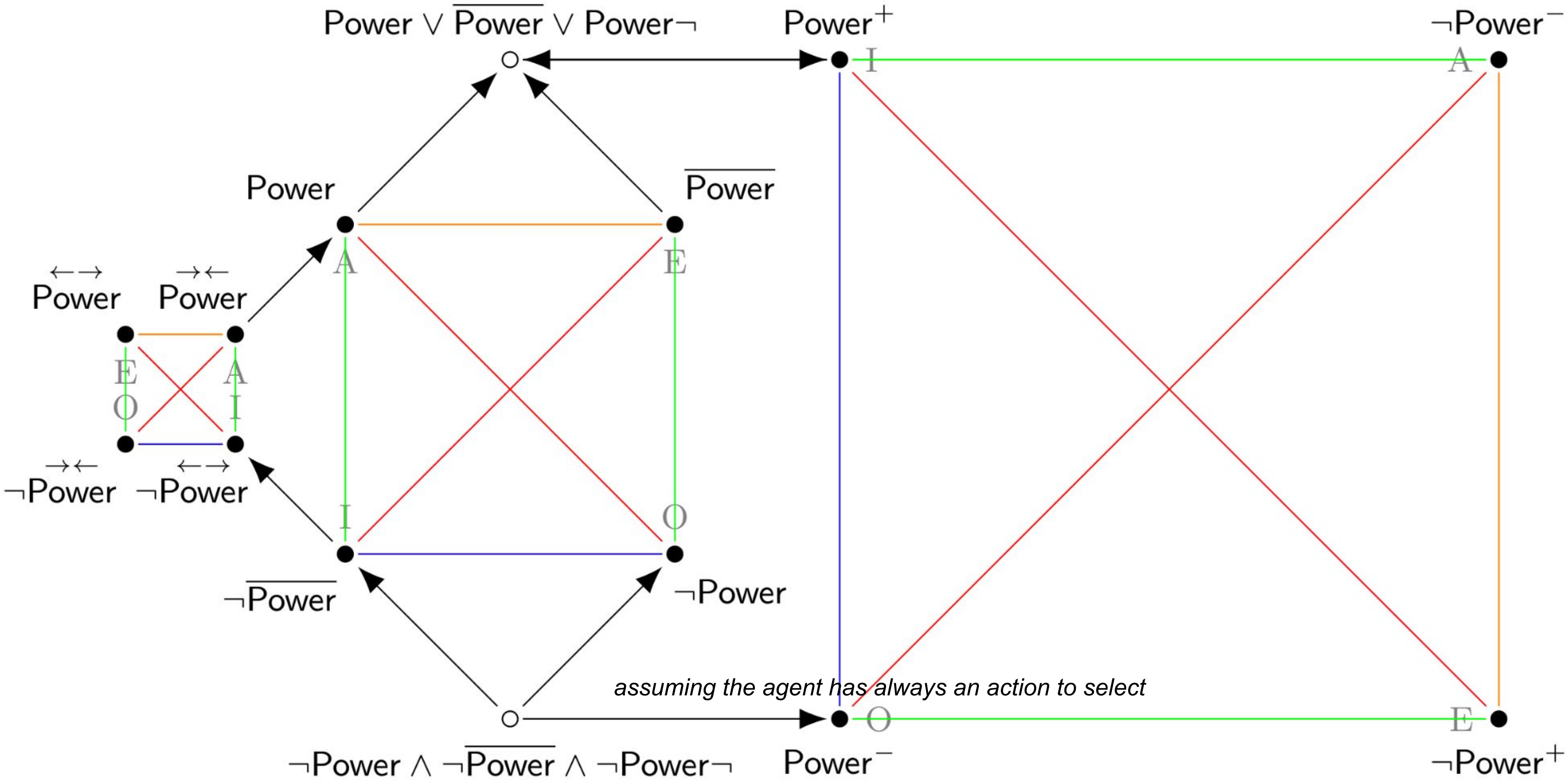


A map of potestative relations

Putting together the three squares for power, and expanding the outcome-centered one to an *Aristotelian hexagon*, we get a complex diagram showing connections between notions of a different family.



$\mathsf{Power} \vee \overline{\mathsf{Power}}$
Power^{+}
$\neg\mathsf{Power}^{-}$
Power
$\overline{\mathsf{Power}}$
$\overset{\leftarrow\rightarrow}{\mathsf{Power}}$
$\overset{\rightarrow\leftarrow}{\mathsf{Power}}$
$\neg\overset{\rightarrow\leftarrow}{\mathsf{Power}}$
$\neg\overset{\leftarrow\rightarrow}{\mathsf{Power}}$
$\neg\overline{\mathsf{Power}}$
$\neg\mathsf{Power}$
$\neg\mathsf{Power} \wedge \neg\overline{\mathsf{Power}}$
Power^{-}
$\neg\mathsf{Power}^{+}$
A
E
I
O



Power ∨ $\overline{\text{Power}}$ ∨ Power¬
Power$^+$
¬Power$^-$
I
A
Power
$\overline{\text{Power}}$
A
E
$\overleftrightarrow{\text{Power}}$
$\overrightarrow{\leftarrow}$Power
E
A
O
I
¬Power
¬Power
I
O
¬$\overline{\text{Power}}$
¬Power
assuming the agent has always an action to select
O
E
¬Power ∧ ¬$\overline{\text{Power}}$ ∧ ¬Power¬
Power$^-$
¬Power$^+$

From squares to hexagons

Moving from Aristotelian squares to Aristotelian hexagons allows one to recover some lost symmetries observed in previous formalizations.

$$\mathsf{NoClaim}(x, y, A) \equiv \neg\mathsf{Claim}(x, y, A)$$
$$\mathsf{Duty}(y, x, A) \equiv \mathsf{Claim}(x, y, A)$$
$$\mathsf{Liberty}(y, x, A) \equiv \neg\mathsf{Claim}(x, y, \overline{A})$$

$$\mathsf{Disability}(x, y, A) \equiv \neg\mathsf{Power}(x, y, A)$$
$$\mathsf{Liability}(y, x, A) \equiv \mathsf{Power}(x, y, A)$$
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asymmetry

$$\mathsf{Disability}(x, y, A) \equiv \neg\mathsf{Power}(x, y, A)$$
$$\mathsf{Liability}(y, x, A) \equiv \mathsf{Power}(x, y, A)$$
$$\mathsf{Immunity}(y, x, A) \equiv \neg\mathsf{Power}(x, y, A)$$

$$\neg\mathsf{Duty}(y, x, A) \equiv \mathsf{Liberty}(y, x, \overline{A})$$

negating a duty of A means having the liberty to <u>omit</u> A

$$\neg\mathsf{Liability}(x, y, A) \equiv \mathsf{Immunity}(x, y, A)$$

negating a liability to A means having an immunity to A

From squares to hexagons

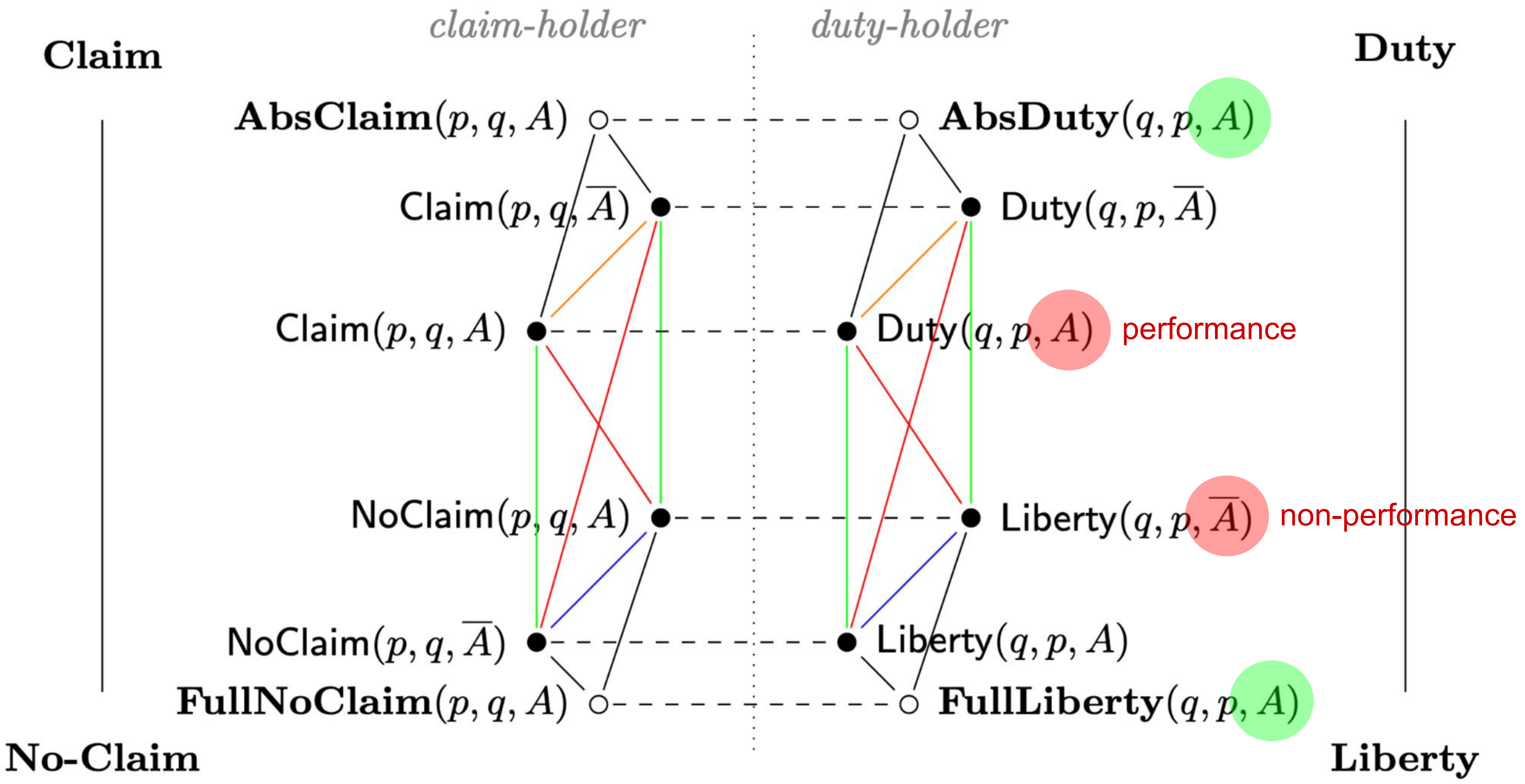
Moving from Aristotelian squares to Aristotelian hexagons allows one to recover some lost symmetries observed in previous formalizations.

A **full-liberty** with respect to an action A means that one has both the possibility of performing A and the possibility of not performing A. An Aristotelian square is able to capture only **half-liberties**.

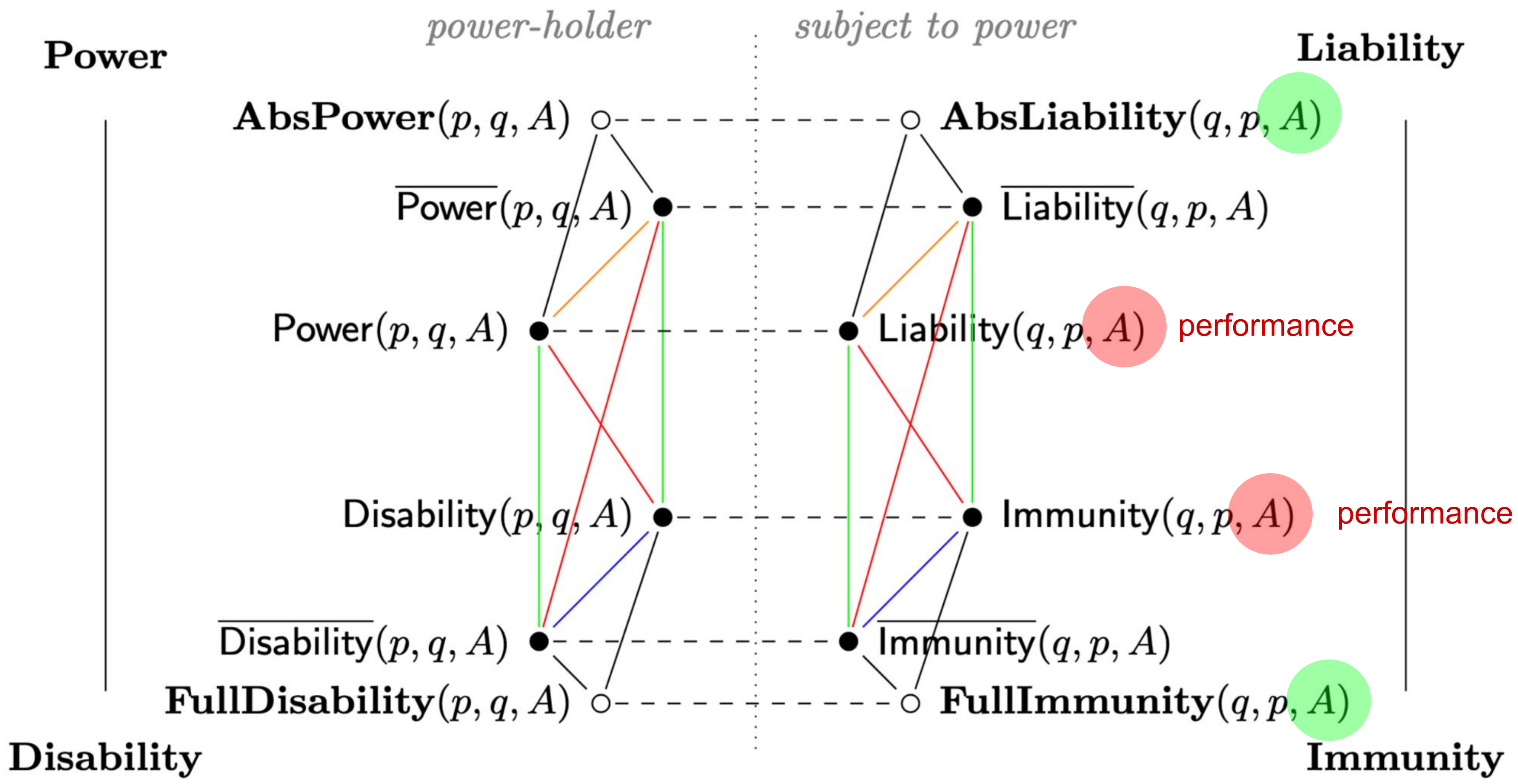
An **absolute duty** with respect to A means that A is the object of a command: one either must perform A or must refrain from A. An Aristotelian square is able to capture only positive-duty (obligation) or negative-duty (prohibition).

An absolute duty is the negation of a full liberty.

The same analysis applies to other first-order and second-order relations.



claim-holder
duty-holder
Claim
Duty
AbsClaim(p, q, A)
AbsDuty(q, p, A)
Claim$(p, q, \overline{A})$
Duty$(q, p, \overline{A})$
Claim(p, q, A)
Duty(q, p, A)
performance
NoClaim(p, q, A)
Liberty$(q, p, \overline{A})$
non-performance
NoClaim$(p, q, \overline{A})$
Liberty(q, p, A)
FullNoClaim(p, q, A)
FullLiberty(q, p, A)
No-Claim
Liberty



power-holder
subject to power
Power
Liability
AbsPower(p, q, A)
AbsLiability(q, p, A)
$\overline{\text{Power}}(p, q, A)$
$\overline{\text{Liability}}(q, p, A)$
Power(p, q, A)
Liability(q, p, A)
performance
Disability(p, q, A)
Immunity(q, p, A)
performance
$\overline{\text{Disability}}(p, q, A)$
$\overline{\text{Immunity}}(q, p, A)$
FullDisability(p, q, A)
FullImmunity(q, p, A)
Disability
Immunity

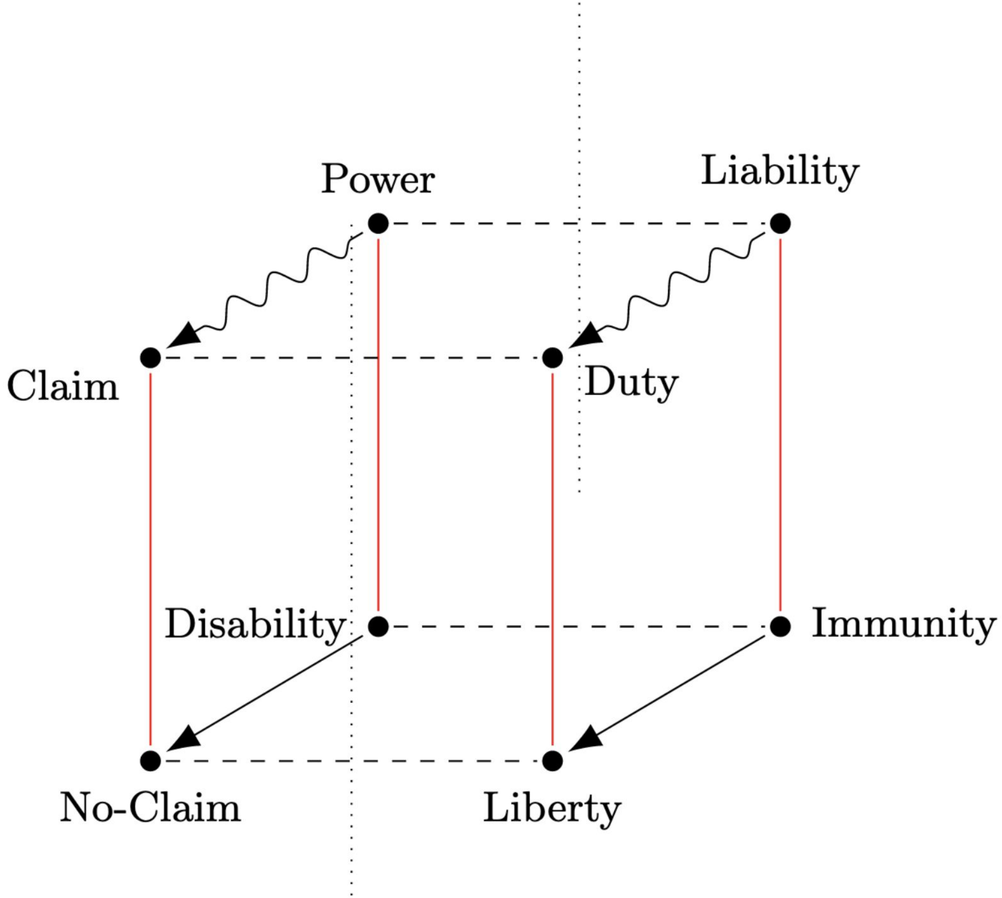
Interactions between the two orders

Andrews (1983) proposed a diagram, called Hohfeld’s cube, to investigate relationships between deontic and potestative concepts. This was a first attempt towards visualizing how the two families of concepts interact.

We have redrawn it according to our conceptualization...

Hohfeld’s cube

- It is expected that at some point power (liability) is transformed into claim (duty).
- An immunity (disability) puts the agent at liberty (no-claim) against one’s power



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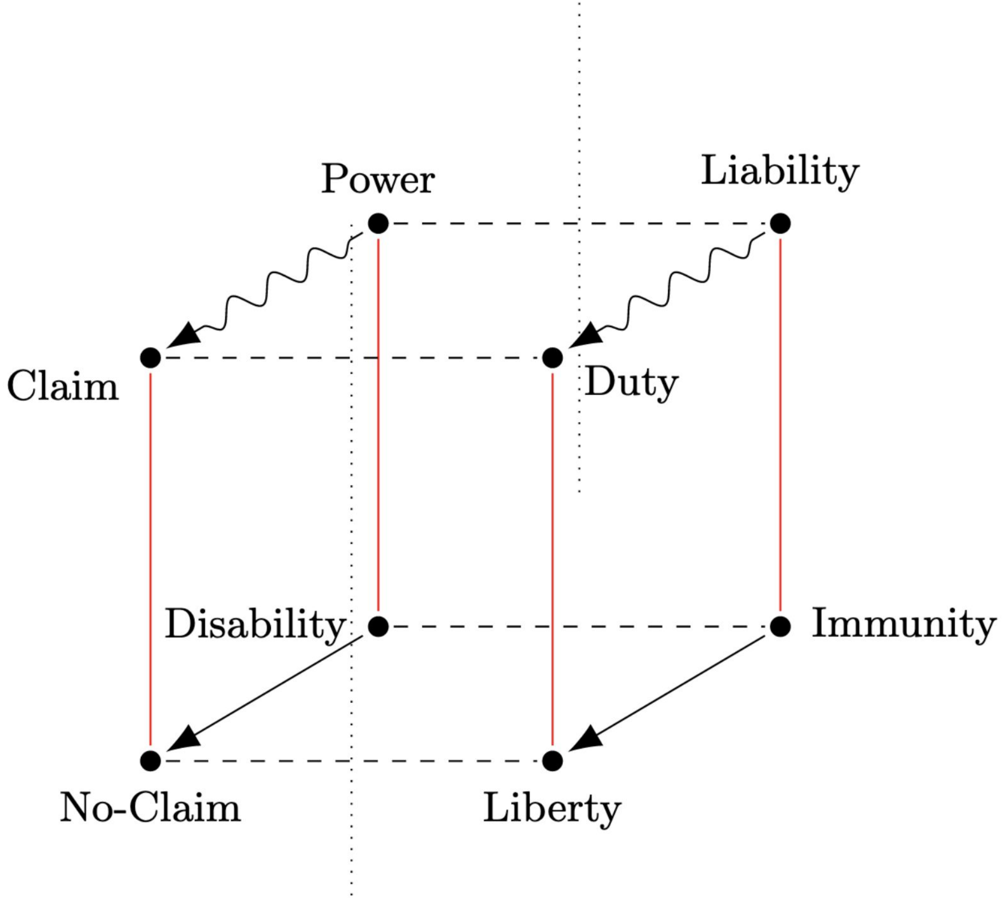

The problem of transforming Hohfeld’s cube into an Aristotelian solid remains open.

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