Free University of Bozen-Bolzano
KRDB Research Centre for Knowledge and Data

A POSSIBILITY-BASED EPISTEMIC PLANNING FRAMEWORK

36th Italian Conference on Computational Logic

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Overview

Chapter 1

Multi-Agent Epistemic Planning

Introduction

Epistemic Reasoning

Reasoning not only about agents' *perception of the world* but also about agents' *knowledge* and/or *beliefs* of her and others' beliefs.

Multi-agent Epistemic Planning Problem [BA11]

Finding *plans* where the goals can refer to:

- the state of the world
- the knowledge and/or the beliefs of the agents

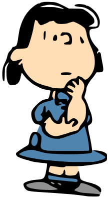
Epistemic State

Epistemic states (e-states) must carry two kinds of information:

- Factual information of multiple possible worlds (atoms)
- Epistemic information: *beliefs* of agents

Heads or Tails?

- Charlie puts a coin in the box while Lucy is not looking
- Only Charlie one knows the coin position



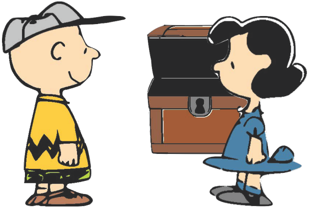
Epistemic Action

Similarly for actions:

- Effects of multiple possible events
- Perspective of agents about the events

Peeking into the box

- Lucy peeks into the box and learns the coin position
- Charlie is aware of it; Lucy is aware that Charlie is aware of it; and so forth



Chapter 2

Epistemic States as Kripke Models

Kripke Models

Definition (Kripke model)

Triple $M = (W, R, V)$ where:

- $W \neq \varnothing$ is the set of possible worlds.
- $R : \mathcal{AG} \rightarrow 2^{W \times W}$ assigns to each agent $\mathtt{i}$ an accessibility relation $R_{\mathtt{i}}$.
- $V : \mathcal{P} \rightarrow 2^{W}$ assigns to each atom a set of worlds.

E-states are represented by *(multi-)pointed Kripke models* (M, W_d), where $W_d \subseteq W$ is a non-empty set of designated worlds.

Event Models

Definition (Event Model)

Quadruple $\mathcal{E} = (E, Q, pre, post)$ where:

- $E \neq \varnothing$ is the set of events, called *domain*.
- $Q : \mathcal{AG} \rightarrow 2^{E \times E}$ assigns to each agent $\mathtt{i}$ an accessibility relation $Q_{\mathtt{i}}$.
- $pre : E \rightarrow \mathcal{L}^{C}_{\mathcal{P},\mathcal{AG}}$ assigns to each event a *precondition*.
- $post : E \rightarrow (\mathcal{P} \rightarrow \mathcal{L}^{C}_{\mathcal{P},\mathcal{AG}})$ assigns to each event a *postcondition* for each atom.

Actions are represented by *(multi-)pointed event models* $(\mathcal{E}, E_d)$, where $E_d \subseteq E$ is a non-empty set of designated events.

Product Update

How do we update an e-state when an action occurs?

Definition (Product Update)

Action $(\mathcal{E}, E_d)$ and e-state (M, W_d). The *product update* is $(M, W_d) \otimes (\mathcal{E}, E_d) = ((W', R', V'), W'_d)$, where:

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$$W' = \{(w, e) \in W \times E \mid (M, w) \models pre(e)\}$$

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$$R'_{\mathtt{i}} = \{((w, e), (v, f)) \in W' \times W' \mid wR_{\mathtt{i}}v \text{ and } eQ_{\mathtt{i}}f\}$$

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V'(\mathtt{p}) &= \{(w, e) \in W' \mid (M, w) \models post(e)(\mathtt{p})\}
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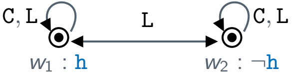
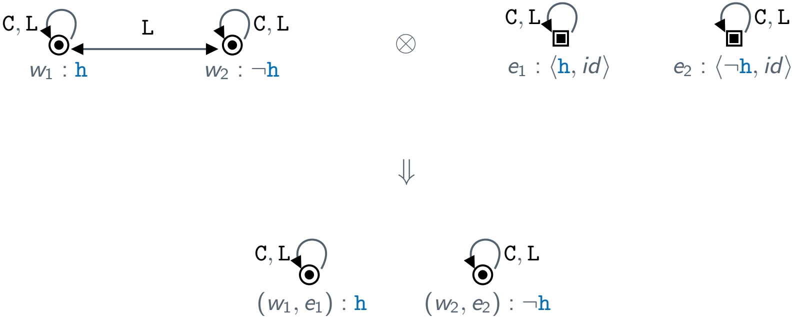
W'_d &= \{(w, e) \in W' \mid w \in W_d \text{ and } e \in E_d\}
\end{aligned}
$$

Example



Example

Example



Chapter 3

Epistemic States as Possibilities

Overview

- Introduced by Gerbrandy and Groeneveld [GG97]
- Based on *non-well-founded sets*
- We use them to represent *both* epistemic states and actions

Why Possibilities?

- More compact representation (wrt Kripke models)
- Faster implementation
- Provide a more fitting intuition for describing state of minds/perspectives of agents
- Tight bond to Kripke models: we can exploit results based on Kripke models

Possibilities

Definition (Possibilities [GG97])

A *possibility* u is a function that assigns to each atom $p \in \mathcal{P}$ a truth value $u(p) \in \{0, 1\}$ and to each agent $i \in \mathcal{AG}$ a set of possibilities $u(i)$.

Intuitively a possibility is a possible configuration of the world:

- $u(p)$ specifies the truth value of the atom p (plays the role of the valuation function)
- $u(i)$ is the set of all the worlds that agent i considers possible in u (plays the role of the accessibility relations)

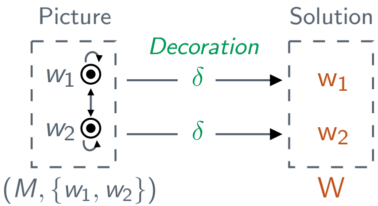
An e-state is represented by a *possibility spectrum* $U = \{u_1, \ldots, u_k\}$, which is a non-empty set of designated possibilities.

From Possibilities to Kripke Models

Definition (Decoration of a Kripke Model)

The *decoration* of a Kripke model $M = (W, R, V)$ is a function δ that, for each $w \in W$, it assigns a possibility $\mathsf{w} = \delta(w)$ such that:

- $\mathsf{w}(\mathtt{p}) = 1$ iff $w \in V(\mathtt{p})$ for each $\mathtt{p} \in \mathcal{P}$, and
- $\mathsf{w}(\mathtt{i}) = \{\delta(w') \mid wR_{\mathtt{i}}w'\}$ for each $\mathtt{i} \in \mathcal{AG}$.

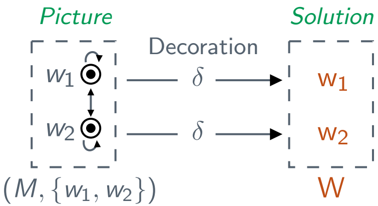


From Possibilities to Kripke Models

Definition (Picture and Solution)

If δ is a decoration of $M = (W, R, V)$ and $W_d \subseteq W$, then:

- (M, W_d) is a *picture* of the possibility spectrum $\mathsf{W} = \{\delta(w) \mid w \in W_d\}$, and
- W is said to be the *solution* of (M, W_d).



Possibility-based Event Models

Let $\mathcal{P}' = \mathcal{P} \cup \{\texttt{pre}\}$, where $\texttt{pre} \notin \mathcal{P}$ is a fresh propositional atom.

Definition (Possibility-based Event Model (PEM))

A *PEM* e is a function that assigns to each atom $\texttt{p'} \in \mathcal{P}'$ a formula $\mathsf{e}(\texttt{p'}) \in \mathcal{L}^{C}_{\mathcal{P},\mathcal{AG}}$ and to each agent $\texttt{i} \in \mathcal{AG}$ a *set of PEMs* $\mathsf{e}(\texttt{i})$.

Intuitively a PEM is a possible interpretation of an action and the perspectives each agent has towards it:

- $\mathsf{e}(\texttt{pre})$ and $\mathsf{e}(\texttt{p})$ ($\texttt{p} \in \mathcal{P}$) specify the pre-/postconditions
- $\mathsf{e}(\texttt{i})$ is the set of all the events that agent $\texttt{i}$ considers possible in e

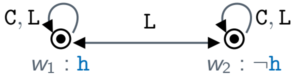
An action is represented by an *event spectrum* $\mathsf{E} = \{\mathsf{e}_1, \ldots, \mathsf{e}_k\}$, which is a non-empty set of designated PEMs.

Decoration, picture and solution are defined similarly.

Example

Possibility spectrum $W = \{w_1, w_2\}$, where:

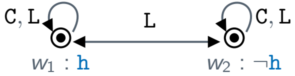
- $w_1(\mathtt{h}) = 1$, $w_2(\mathtt{h}) = 0$;
- $w_1(\mathtt{C}) = \{w_1\}$, $w_2(\mathtt{C}) = \{w_2\}$, $w_1(\mathtt{L}) = w_2(\mathtt{L}) = \{w_1, w_2\}$.



Example

Possibility spectrum $\mathsf{W} = \{\mathsf{w}_1, \mathsf{w}_2\}$, where:

- $\mathsf{w}_1(\mathtt{h}) = 1$, $\mathsf{w}_2(\mathtt{h}) = 0$;
- $\mathsf{w}_1(\mathtt{C}) = \{\mathsf{w}_1\}$, $\mathsf{w}_2(\mathtt{C}) = \{\mathsf{w}_2\}$, $\mathsf{w}_1(\mathtt{L}) = \mathsf{w}_2(\mathtt{L}) = \{\mathsf{w}_1, \mathsf{w}_2\}$.



Event spectrum $\mathsf{E} = \{\mathsf{e}_1, \mathsf{e}_2\}$, where:

- $\mathsf{e}_1(\mathtt{pre}) = \mathtt{h}$, $\mathsf{e}_2(\mathtt{pre}) = \neg\mathtt{h}$ and $\mathsf{e}_1(\mathtt{h}) = \mathsf{e}_2(\mathtt{h}) = \mathtt{h}$;
- $\mathsf{e}_1(\mathtt{C}) = \mathsf{e}_1(\mathtt{L}) = \{\mathsf{e}_1\}$ and $\mathsf{e}_2(\mathtt{C}) = \mathsf{e}_2(\mathtt{L}) = \{\mathsf{e}_2\}$.

A Quick Recap

- A *possibility* represents a possible world (atoms + beliefs) → A *possibility spectrum* represents an e-state
- A *PEM* represents an event (pre-/postconditions + beliefs) → An *event spectrum* represents an action

Chapter 4

DELPHIC

A New Framework for Epistemic Planning

DEL-planning with a **P**ossibility-based **H**omogeneous **I**nformation **C**haracterisation

- E-states and actions are represented using possibilities
- New element: *union update* (update operator)

Union Update

Definition (Union Update)

The *union update* of a possibility u with a PEM e is the possibility $u' = u \uplus e$, such that if $u \not\models e(\mathtt{pre})$, then $u' = \varnothing$; otherwise:

$$u'(p) = 1 \text{ iff } u \models e(p)$$

$$u'(\mathtt{i}) = \{v \uplus f \mid v \in u(\mathtt{i}), f \in e(\mathtt{i}) \text{ and } v \models f(\mathtt{pre})\}$$

Union Update

Definition (Union Update)

The *union update* of a possibility u with a PEM e is the possibility $u' = u \uplus e$, such that if $u \not\models e(\mathtt{pre})$, then $u' = \emptyset$; otherwise:

$$u'(\mathtt{p}) = 1 \text{ iff } u \models e(\mathtt{p})$$
$$u'(\mathtt{i}) = \{v \uplus f \mid v \in u(\mathtt{i}), f \in e(\mathtt{i}) \text{ and } v \models f(\mathtt{pre})\}$$

The *union update* of a possibility spectrum U with an event spectrum E is:

$$U \uplus E = \{u \uplus e \mid u \in U, e \in E \text{ and } u \models e(\mathtt{pre})\}.$$

Example

Solution: $W = \{w_1, w_2\}$ Solution: $E = \{e_1, e_2\}$

$W \uplus E = \{w_x^y \mid w_x \in W, e_y \in E$ and $w_x \models e_y(\texttt{pre})\}$, where $w_x^y = w_x \uplus e_y$.

Example

Solution: $W = \{w_1, w_2\}$

Solution: $E = \{e_1, e_2\}$

$W \uplus E = \{w_x^y \mid w_x \in W, e_y \in E \text{ and } w_x \models e_y(\mathtt{pre})\}$, where $w_x^y = w_x \uplus e_y$.

Since $w_1 \uplus e_2 = w_2 \uplus e_1 = \varnothing$, we have $W \uplus E = \{w_1^1, w_2^2\}$, where:

- $w_1^1(\mathtt{h}) = 1$, $w_2^2(\mathtt{h}) = 0$.

Example

Solution: $W = \{w_1, w_2\}$

Solution: $E = \{e_1, e_2\}$

$W \otimes E = \{w_x^y \mid w_x \in W, e_y \in E \text{ and } w_x \models e_y(\mathtt{pre})\}$, where $w_x^y = w_x \otimes e_y$.

Since $w_1 \otimes e_2 = w_2 \otimes e_1 = \emptyset$, we have $W \otimes E = \{w_1^1, w_2^2\}$, where:

- $w_1^1(\mathtt{h}) = 1$, $w_2^2(\mathtt{h}) = 0$.
- $w_1^1(\mathtt{C}) = \{w_x^y \mid w_x \in w_1(\mathtt{C}), e_y \in e_1(\mathtt{C}) \text{ and } w_x \models e_y(\mathtt{pre})\} = \{w_1^1\}$

Example

Solution: $W = \{w_1, w_2\}$ Solution: $E = \{e_1, e_2\}$

$W \mathbin{\cup\!\!\!\!\!\times} E = \{w_x^y \mid w_x \in W, e_y \in E \text{ and } w_x \models e_y(\texttt{pre})\}$, where $w_x^y = w_x \mathbin{\cup\!\!\!\!\!\times} e_y$.

Since $w_1 \mathbin{\cup\!\!\!\!\!\times} e_2 = w_2 \mathbin{\cup\!\!\!\!\!\times} e_1 = \varnothing$, we have $W \mathbin{\cup\!\!\!\!\!\times} E = \{w_1^1, w_2^2\}$, where:

- $w_1^1(\texttt{h}) = 1$, $w_2^2(\texttt{h}) = 0$.
- $w_1^1(\texttt{C}) = \{w_x^y \mid w_x \in w_1(\texttt{C}), e_y \in e_1(\texttt{C}) \text{ and } w_x \models e_y(\texttt{pre})\} = \{w_1^1\}$; $w_1^1(\texttt{L}) = \{w_1^1\}$

Example

Solution: $W = \{w_1, w_2\}$

Solution: $E = \{e_1, e_2\}$

$W \uplus E = \{w_x^y \mid w_x \in W, e_y \in E \text{ and } w_x \models e_y(\mathtt{pre})\}$, where $w_x^y = w_x \uplus e_y$.

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- $w_1^1(\mathtt{C}) = w_1^1(\mathtt{L}) = \{w_1^1\}$

Example

Solution: $W = \{w_1, w_2\}$

Solution: $E = \{e_1, e_2\}$

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Example

Solution: $W = \{w_1, w_2\}$

Solution: $E = \{e_1, e_2\}$

$W \ltimes E = \{w_x^y \mid w_x \in W, e_y \in E \text{ and } w_x \models e_y(\mathtt{pre})\}$, where $w_x^y = w_x \ltimes e_y$.

Since $w_1 \ltimes e_2 = w_2 \ltimes e_1 = \varnothing$, we have $W \ltimes E = \{w_1^1, w_2^2\}$, where:

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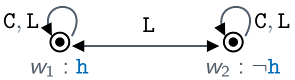
Example



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$\uplus$

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Since $w_1 \uplus e_2 = w_2 \uplus e_1 = \emptyset$, we have $W \uplus E = \{w_1^1, w_2^2\}$, where:

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Example

$\mathsf{W}' = \mathsf{W} \uplus \mathsf{E} = \{\mathsf{w}_1^1, \mathsf{w}_2^2\}$, where:

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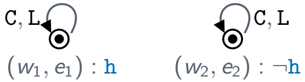


$(M', \{w_1^1, w_1^2\})$

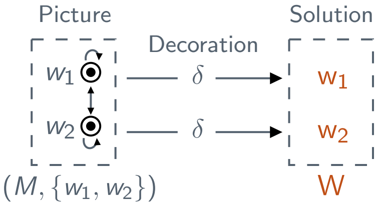
Observation

W' is the *solution* of $(M', \{w_1^1, w_1^2\})$!

Update Equivalence

Theorem

Let $(\mathcal{E}, E_d)$ be an action applicable in an e-state (M, W_d), with solutions E *and* W*, respectively. Then the possibility spectrum* $\mathsf{W}' = \mathsf{W} \uplus \mathsf{E}$ *is the solution of* $(M', W'_d) = (M, W_d) \otimes (\mathcal{E}, E_d)$.



Chapter 5

Conclusions

Conclusions

- We provided a new framework for epistemic planning which is entirely based on possibilities
- Motivated by previous implementations based on possibilities
- More compact representation
- Semantical equivalence with the Kripke-based formalism

Future works

- We are currently implementing DELPHIC within the planner EFP
- We will be able to handle user-provided actions (high level of customisation)
- General framework: wide variety of real-world scenarios

The end

Thank You
for the attention