University of Udine Department of Mathematics, Computer Science and Physics

## MODELLING MULTI-AGENT EPISTEMIC PLANNING IN ASP



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- 1. Multi-Agent Epistemic Planning
- 2. Possibilities
- 3. The action language  $m\mathcal{A}^{
  ho}$
- 4. PLATO
- 5. Conclusions

Chapter 1

# Multi-Agent Epistemic Planning



#### **Epistemic Reasoning**

Reasoning not only about agents' *perception of the world* but also about agents' *knowledge* and/or *beliefs* of her and others' beliefs.

#### Multi-agent Epistemic Planning Problem [BA11]

Finding *plans* where the goals can refer to:

- the state of the world
- the knowledge and/or the beliefs of the agents



#### Initial State

- Snoopy and Charlie are looking while Lucy is -looking
- No one knows the coin position.



Multi-Agent Epistemic Planning **An Example** 



#### Goal State

- Charlie knows the coin position
- Lucy knows that Charlie knows the coin position
- Snoopy does not know anything about the plan execution



#### Given a set of <code>agents</code> $\mathcal{AG}$

#### Belief formulae

where  $\mathtt{ag} \in \mathcal{AG}$  ,  $\alpha \subseteq \mathcal{AG}$ 

We use the operators  $\mathsf{B}_{\mathsf{ag}}$  and  $\mathsf{C}_\alpha$  to model the belief and the common belief of the <code>agents</code>.

# Properties of $B_{ag}$ KD45, and S5, AxiomsGiven the fluent formulae $\phi$ , $\psi$ and the worlds i, jD $\neg \mathcal{R}_i \bot$ B $\mathcal{K}$ K $(\mathcal{R}_i \varphi \land \mathcal{R}_i (\varphi \Longrightarrow \psi)) \Longrightarrow \mathcal{R}_i \psi$ T $\mathcal{R}_i \varphi \Longrightarrow \varphi$ K $\mathcal{R}_i \varphi \Longrightarrow \varphi$ K $\mathcal{R}_i \varphi \Longrightarrow \mathcal{R}_i \mathcal{R}_i \varphi$ S $\neg \mathcal{R}_i \varphi \Longrightarrow \mathcal{R}_i \neg \mathcal{R}_i \varphi$

Chapter 2

# Possibilities



Overview

- Introduced by Gerbrandy and Groeneveld [GG97]
- Used to represent multi-agent information change
- Based on non-well-founded sets



Let  $\mathcal{AG}$  be a set of agents and  $\mathcal F$  a set of fluents.

#### Possibility [GG97]

A *possibility* u is a function that assigns to each fluent  $\mathbf{f} \in \mathcal{F}$  a truth value  $u(\mathbf{f}) \in \{0, 1\}$  and to each agent  $\mathbf{ag} \in \mathcal{AG}$  a set of possibilities  $u(\mathbf{ag}) = \sigma$  (*information state*).

Intuitively a possibility is a possible configuration of the world:

- u(f) specifies the truth value of the fluent f (plays the role of the valuation function)
- u(ag) is the set of all the worlds that agent ag considers possible in u (plays the role of the accessibility relations)
- Representable with graphs: we will use graph terminology

Chapter 3

# The action language $m\mathcal{A}^{\rho}$



We introduced the action language  $m\mathcal{A}^{
ho}$  in [Fab+20]

- Used to describe MEP problems
- Uses possibilities as states
- Actions preconditions: belief formulae



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- Three types of actions:
  - Ontic: modifies some fluents of the world Charlie opens the box





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- Three types of actions:
  - Ontic: modifies some fluents of the world Charlie *opens* the box
  - Sensing: senses the true value of a fluent Charlie peeks inside the box





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#### Three types of actions:

- Ontic: modifies some fluents of the world
- Sensing: senses the true value of a fluent Charlie *peeks* inside the box
- Announcement: announces the fluent to other agents Charlie announces the coin position

#### The action language $m\mathcal{A}^{\rho}$ Action types





# **Observability Relations**



# An *execution* of an action might change or not an agents' belief accordingly to her degree of awareness

Action type	Full observers	Partial Observers	Oblivious
Ontic	$\checkmark$		$\checkmark$
Sensing	$\checkmark$	$\checkmark$	$\checkmark$
Announcement	$\checkmark$	$\checkmark$	$\checkmark$

Chapter 4

# PLATO



#### PLATO, ePistemic muLti-agent Answer seT programming sOlver:

- Declarative encoding in ASP of MEP
- Based on the language  $m\mathcal{A}^{\rho}$
- Main components: *initial state generation*, *entailment*, *transition function*
- Exploits *clingo*'s multi-shot capabilities [Geb+19]
- Formal proof of correctness

#### PLATO ASP Encoding

# **Encoding possibilities**

Let u be a possibility.

#### ASP encoding: possibilities

We encode u with the atom  $possible_world(T_u, R_u, P_u)$ , where:

- $T_{\mathrm{u}}$  tells us when u was created
- $R_u$  is the *repetition* of u
- $\ensuremath{\mathtt{P}}_u$  is the numerical index of u

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#### ASP encoding: pointed possibility

If u is the possibility that represents the *real configuration* of the world, we encode it with the atom  $pointed(T_u, R_u, P_u)$ .

When the context is clear we will use  $\textit{only} \ P_u$  instead of  $(T_u, R_u, P_u).$ 

# **Encoding possibilities**



Let u, v be two possibilities, let AG be an agent and let F be a fluent.

#### ASP encoding: information states

We encode  $v \in u(AG)$  with the atom  $believes(P_u, P_v, AG)$ .

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#### ASP encoding: interpretations

We encode u(F) = 1 with the atom  $holds(P_u, F)$ .





entails	(P,	F)	:- holds(P,F), fluent(F).
entails	(P,	neg(F))	:- not entails(P,F).
entails	(P,	and(F1,F2))	:- entails(P,F1), entails(P,F2).
entails	(P,	or(F1,F2))	:- entails(P,F1).
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entails	(P,	or(F1, F2))	:- entails(P,F2).
not_entails	(P1,	b(AG,F))	:- not entails(P2,F), believes(P1,P2,AG).
entails	(P,	b(AG,F))	:- not not_entails(P,b(AG,F)).



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entails	(P,	or(F1, F2))	:- entails(P,F1).
entails	(P,	or(F1,F2))	:- entails(P,F2).
not_entails	(P1,	b(AG,F))	:- not entails(P2,F), believes(P1,P2,AG).
entails	(P,	b(AG,F))	$:= not not_entails(P, b(AG, F)).$
not_entails	(P1,	c(AGS,F))	:- not entails(P2,F), reaches(P1,P2,AGS).
entails	(P,	c(AGS, F))	$:- not not_entails(P, c(AGS, F)).$

Let open be an ontic action such that

- It sets the fluent opened to true
- Only Charlie and Lucy are attentive





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 $u_1(\mathcal{F}) = \{\text{head}, \text{looking}_{\text{Charlie}}, \text{looking}_{\text{Lucy}}\}$  $u_2(\mathcal{F}) = \{\text{looking}_{\text{Charlie}}, \text{looking}_{\text{Lucy}}\}$ 

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 $pw(u'_i) := pointed(u_1), reaches(u_1, u_i, AGS), fully_obs(AGS).$   $(i \in \{1, 2\})$ 



$$\begin{split} u_1(\mathcal{F}) &= \{\text{head}, \text{looking}_{\text{charlie}}, \text{looking}_{\text{Lucy}} \} \\ u_2(\mathcal{F}) &= \{\text{looking}_{\text{charlie}}, \text{looking}_{\text{Lucy}} \} \\ u_1'(\mathcal{F}) &= \{\text{opened}, \text{head}, \text{looking}_{\text{charlie}}, \text{looking}_{\text{Lucy}} \} \\ u_2'(\mathcal{F}) &= \{\text{opened}, \text{looking}_{\text{charlie}}, \text{looking}_{\text{Lucy}} \} \end{split}$$



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 $\texttt{believes}(\textbf{u}'_i, \textbf{u}'_i, \texttt{AG}) := \texttt{pw}(\textbf{u}'_i), \texttt{pw}(\textbf{u}'_i), \texttt{fully_obs}(\texttt{AG}),$  $(i, j \in \{1, 2\})$  $believes(u_i, u_i, AG), pw(u_i), pw(u_i).$ 



- ) C,L,S  $u_1(\mathcal{F}) = \{\text{head}, \text{looking}_{\text{Charlie}}, \text{looking}_{\text{ulg}}\}$  $u_2(\mathcal{F}) = \{\text{looking}_{\text{Charlie}}, \text{looking}_{\text{Lucy}}\}$  $u_1(\mathcal{F}) = \{\text{head}, \text{looking}_{\text{Charlie}}, \text{looking}_{\text{Lucv}}\}$ 

  - $u'_1(\mathcal{F}) = \{\text{opened}, \text{head}, \text{looking}_{\text{Charlie}}, \text{looking}_{\text{Lucy}}\}$
  - $u_2'(\mathcal{F}) = \{\text{opened}, \text{looking}_{\text{Charlie}}, \text{looking}_{\text{Lucy}}\}$



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Let *peek* be an ontic action such that

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 $pw(u_2'') := pointed(u_1'), reaches(u_1', u_2', AGS), not_oblivious(AGS).$ 



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 $\begin{array}{lll} \texttt{believes}(\textbf{u}_i'',\textbf{u}_j'',\texttt{AG}):&=&\texttt{pw}(\textbf{u}_i''),\texttt{pw}(\textbf{u}_j'),\texttt{pw}(\textbf{u}_i'),\texttt{pw}(\textbf{u}_j'), \quad (i,j\in\{1,2\})\\ &&\texttt{believes}(\textbf{u}_i',\textbf{u}_j',\texttt{AG}),\texttt{partially\_obs}(\texttt{AG}). \end{array}$ 



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Let  $\mathsf{u},\mathsf{v}$  be two possibilities and  $\psi$  be a belief formula.

#### Entailment correctness

For each u, we have that  $\forall \ \psi \ \mathsf{u} \models_{\Phi} \psi$  iff  $\mathsf{u} \models_{\Gamma} \psi$  .

#### Initial state generation correctness

For each u, v such that u is the initial state in  $m\mathcal{A}^{\rho}$  and v is the initial state in PLATO then  $\forall \psi \ u \models_{\Phi} \psi$  iff  $v \models_{\Gamma} \psi$ .

#### Transition function correctness

Let a be an action instance. For each u, v such that  $\forall \psi \ u \models_{\Phi} \psi$ iff  $v \models_{\Gamma} \psi$ , then  $\forall \psi \ \Phi(a, u) \models_{\Phi} \psi$  iff  $\Gamma(a, v) \models_{\Gamma} \psi$ .

#### PLATO Results

## **Experimental evaluation**



	<b>SC</b> : $ \mathcal{AG}  = 9,  \mathcal{F}  = 12,  \mathcal{A}  = 14$								
L many frumpy K-BIS P-MAR									
4	.24	.24	.03	.007					
6	2.56	2.49	.16	.04					
8	36.79	38.34	4.23	.30					
9	204.52	146.343	5.79	.83					
10	TO	839.27	7.36	1.78					

$\mathbf{Gr:} \  \mathcal{AG}  = 3, \  \mathcal{F}  = 9, \  \mathcal{A}  = 24$									
L	Total	Ground	Solve	Atoms					
3	.97	.60	.36	28'615					
4	4.25	2.24	2.01	42'022					
5	32.83	2.52	30.31	71'482					
6	211.69	5.27	206.41	140'305					
7	1066.80	16.94	1049.86	302'623					

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L	Total	Ground	Solve	Atoms				
$     \begin{array}{c}       3 \\       4 \\       5 \\       6 \\       7     \end{array} $	.97 4.25 32.83 211.69 1066.80	.60 2.24 2.52 5.27 16.94	.36 2.01 30.31 206.41 1049.86	28'615 42'022 71'482 140'305 302'623				

	<b>CC</b> <sub>-1</sub> : $ \mathcal{AG}  = 2,  \mathcal{F}  = 10,  \mathcal{A}  = 16$				<b>CC</b> _2:	$\mathcal{AG} =3,$	$ \mathcal{F}  = 13,$	$ \mathcal{A}  = 24$
L	single	multi	K-BIS	P-MAR	single	multi	K-BIS	P-MAR
3	48.74	6.52	.08	.02	153.76	14.07	.13	.03
4	188.32	8.74	.16	.03	ТО	28.02	.54	.10
5	TO	7.68	1.14	.16	ТО	16.13	4.89	.60
6	1222.67	10.83	4.42	0.64	ТО	14.84	12.66	1.71
7	TO	30.08	16.06	2.61	TO	56.48	142.06	12.37

Chapter 5

# Conclusions





- Exploited a *declarative* approach to implement Multi-Agent Epistemic Planning
- Improved readability and code maintenance
- Straightforward semantical adaptations
- Results comparable to the imperative approach
- Formal *proof* of correctness



Future works

- Enhancement of the entailment rules (  $\checkmark$  )
- Formal proof of equivalence between  $m\mathcal{A}^*$  and  $m\mathcal{A}^\rho~(\checkmark)$
- Bringing Delphic into PLATO ( $\circlearrowleft$ )
- We are using PLATO to implement novel concepts in MEP, such as *trust*, *lies* and *misconceptions*

Conclusions Q&A
The end





# Thank You for the attention

- [BA11] Thomas Bolander and Mikkel Birkegaard Andersen. "Epistemic planning for single-and multi-agent systems". In: Journal of Applied Non-Classical Logics 21.1 (2011), pp. 9–34.
- [Fab+20] Francesco Fabiano et al. "EFP 2.0: A Multi-Agent Epistemic Solver with Multiple E-State Representations". In: Proceedings of the Thirtieth International Conference on Automated Planning and Scheduling, Nancy, France, October 26-30, 2020. AAAI Press, 2020, pp. 101–109. URL: https://aaai.org/ ojs/index.php/ICAPS/article/view/6650.

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- [Geb+19] Martin Gebser et al. "Multi-shot ASP solving with clingo". In: Theory and Practice of Logic Programming 19 (2019), pp. 27–82. DOI: 10.1017/S1471068418000054.
- [GG97] J. Gerbrandy and W. Groeneveld. "Reasoning about information change". In: Journal of Logic, Language and Information 6.2 (1997), pp. 147–169. DOI: 10.1023/A:1008222603071.
- [Son+14] Tran Cao Son et al. "Finitary S5-theories". In: European Workshop on Logics in Artificial Intelligence. Springer. 2014, pp. 239–252.

- Assembly Line (AL): two agents are responsible for processing a different part of a product. They can fail in processing their part and inform the other of the status of her task. The agents decide to assemble the product or restart. Goal: the agents must assemble the product. We change the depth of the belief formulae.
- Coin in the Box (CB). n ≥ 3 agents are in a room. There is a closed box containing a coin. None of the agents know the coin position. One agent has the key. An agent may look inside the box to sense the state of the coin and also share the result.

# Backup Slides Domains II

- Collaboration and Communication (CC). n ≥ 2 agents move along a corridor with k ≥ 2 rooms in which m ≥ 1 boxes can be located. Agents can determine if a certain box is in the room they are in. They can communicate information about the boxes' position. Agents may move only to adjacent rooms.
- ► Grapevine. n ≥ 2 agents are located in k ≥ 2 rooms. Each agent ag knows a "secret" (s\_ag). Agents can move to an adjacent room and share their secret within the same room.
- ► Selective Communication (SC). n ≥ 2 agents within one of the k ≥ 2 rooms in a corridor. Agents can move to an adjacent room. In only one of the rooms, agents may acquire some information q and may communicate it to others.

#### Finitary S5-theory [Son+14]

Let  $\phi$  be a fluent formula and let  $i \in AG$  be an agent. A *finitary* S5-*theory* is a collection of formulae of the form:

(*i*)  $\phi$  (*ii*)  $C \phi$  (*iii*)  $C (B_i \phi \lor B_i \neg \phi)$  (*iv*)  $C (\neg B_i \phi \land \neg B_i \neg \phi)$ 

Each fluent  $f \in \mathcal{F}$  must appear in at least one of the formulae (*ii*)–(*iv*) (for at least one agent  $i \in AG$ ).

A finitary S5-theory has *finitely many* S5-models up to equivalence.

#### Given

- $\mathcal{AG} = \{\texttt{Snoopy}, \texttt{Charlie}, \texttt{Lucy}\}$
- $\mathcal{F}_{-} = \{\texttt{opened}, \texttt{head}, \texttt{looking}_{\texttt{ag}} \} \texttt{ag} \in \mathcal{AG}$



#### Given

- $\mathcal{AG} = \{\texttt{Snoopy}, \texttt{Charlie}, \texttt{Lucy}\}$
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Consider a formula of a finitary **S5** theory.



Formula type:

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Consider a formula of a finitary **S5** theory.



Formula type: (i)  $\phi$ 

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#### Consider a formula of a finitary **S5** theory.



Formula type: (i)  $\phi$ (ii)  $C \phi$ (iii)  $C (B_i \phi \lor B_i \neg \phi)$ 

```
Formula:

C(B_{Lucy}head \lor B_{Lucy} \neg head)
```

#### Given

- $\mathcal{AG} = \{\texttt{Snoopy}, \texttt{Charlie}, \texttt{Lucy}\}$
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Consider a formula of a finitary **S5** theory.



Formula type: (i)  $\phi$ (ii)  $C \phi$ (iii)  $C (B_i \phi \lor B_i \neg \phi)$ (iv)  $C (\neg B_i \phi \land \neg B_i \neg \phi)$ 

#### Definition (Decoration of a Kripke Model)

The *decoration* of a Kripke model M = (W, R, V) is a function  $\delta$  that, for each  $w \in W$ , it assigns a possibility  $\mathbf{w} = \delta(w)$  such that:

-  $\mathsf{w}(\mathtt{f}) = 1$  iff  $w \in V(\mathtt{f})$  for each  $\mathtt{f} \in \mathcal{F}$ , and

- 
$$\mathbf{w}(\mathbf{i}) = \{\delta(w') \mid wR_{\mathbf{i}}w'\}$$
 for each  $\mathbf{i} \in \mathcal{AG}$ .



#### Definition (Picture and Solution)

If  $\delta$  is a decoration of M = (W, R, V) and  $W_d \subseteq W$ , then:

- $(M, W_d)$  is a *picture* of the possibility spectrum  $W = \{\delta(w) \mid w \in W_d\}$ , and
- W is said to be the *solution* of  $(M, W_d)$ .

