A framework for a modular multi-concept lexicographic closure semantics (an abriged report) [1]

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Aim of the talk

We define a *modular multi-concept extension of the lexicographic closure semantics* for defeasible description logics with typicality.

The idea is that of *distributing the defeasible properties of concepts into different* modules, according to their *subject* (a *concept*), and

- defining *a notion of preference for each module/concept* based on the lexicographic closure semantics;
- defining the semantics of the knowledge base by *combining of the multiple preferences* into a single preference.

Motivations

This multi-preferential approach provides a *spectrum of alternative semantics* depending on:

- *the granularity of modularization* (modules containing all conditionals vs. modules containing one conditional);
- the *choice of the semantics for modules* (here lexicographic closure semantics, but any *ranked semantics* could be adopted);
- *how multiple preferences are combined* into a single one.

This approach leads to a *preferential semantics* of the KB and a notion of *conditional entailment* which:

- *satisfies the KLM properties* of system **P**;
- is not subject to *"blockage of property inheritance"*;
- deals properly with *" ambiguity preservation"* [Geffner and Pearl 92].

The Description Logic $\mathcal{ALC}$

Language of $\mathcal{ALC}$

Let $N_C,\ N_R,\ N_I$ be the set of concept names, role names and individual names.

$\mathcal{ALC}$ *concepts*:

$$C := A \mid \top \mid \bot \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \forall S.C \mid \exists S.C$$

where $A \in N_C$, $R \in N_R$

a first order logic or
a polymodal logic (Schild,1991)

A Knowledge Base is a pair KB = (TBox, ABox):

- TBox contains a finite set of *inclusion axioms* $C \sqsubseteq D$

$$Mother\_of\_a\_Doctor \sqsubseteq \exists hasChild.Doctor.$$

- ABox is a set of *individual assertions* of the form $C(a)$ and $R(a,b)$, where $a, b \in N_I$, a set of individual names.
  For instance:

$$Female(mary), \quad hasFriend(mary, carlo)$$
$$(Italian \sqcap \exists hasFriend.Engineer)(carlo)$$

$\mathcal{ALC}$ Semantics

An $\mathcal{ALC}$ interpretation is any structure $I = (\Delta^I, \cdot^I)$ where:

- Δ^I is a domain;
- $\cdot^I$ is an interpretation function that maps
  - each concept name A to set $A^I \subseteq \Delta^I$,
  - each role name R to a binary relation $R^I \subseteq \Delta^I \times \Delta^I$,
  - each individual name a to an element $a^I \in \Delta^I$.
- $\cdot^I$ is extended to complex concepts as follows:
  - $\top^I = \Delta$ $\quad \bot^I = \emptyset$ $\quad (\neg C)^I = \Delta - C^I$
  - $(C \sqcap D)^I = C^I \cap D^I$ $\quad (C \sqcup D)^I = C^I \cup D^I$
  - $(\exists R.C)^I = \{x \in \Delta \mid \exists y.(x, y) \in R^I \text{ and } y \in C^I\}$
  - $(\forall R.C)^I = \{x \in \Delta \mid \forall y.(x, y) \in R^I \text{ implies } y \in C^I\}$

Satisfiability

An interpretation $\mathcal{M} = \langle \Delta, \cdot^I \rangle$ satisfies:

- a concept inclusion axiom $C \sqsubseteq D$ if $C^I \subseteq D^I$;
- an individual assertion $C(a)$ if $a^I \in C^I$;
- an individual assertion $R(a, b)$ if $(a^I, b^I) \in R^I$

Preferential extensions of DLs

- Preferential extensions of description logics allow defeasible inclusions in the knowledge base to model typical properties of individuals. Kraus Lehmann and Magidor's conditional assertions $C \mid\sim D$ become, for $\mathcal{ALC}$:
  - typicality inclusions $\mathbf{T}(C) \sqsubseteq D$ (Giordano et al., LPAR 2007, FI 2009) based on the preferential semantics [KLM 90];
  - defeasible inclusions $C \sqsubseteq\!\!\sim D$ (Britz et al. KR 2008) based on the rational semantics [LM 92].

$\mathcal{ALC}$ with typicality

Preferential Interpretations

A preferential interpretation is a structure $\langle \Delta, <, \cdot^I \rangle$ where:

- Δ and $\cdot^I$ are a domain and an interpretation function, as in $\mathcal{ALC}$ interpretations;
- $<$ is an irreflexive and transitive relation over Δ and is *well-founded*.
  Basic idea: $x < y$ means: x is more normal than y
- $(\mathbf{T}(C))^I = Min_<(C^I)$
- $\mathcal{M} \models \mathbf{T}(C) \sqsubseteq D$ iff $(\mathbf{T}(C))^I \subseteq D^I$

Ranked interpretations

modularity: for all $x, y, z \in \Delta$, if $x < y$ then either $x < z$ or $z < y$
Each $x \in \Delta$ has a rank $k_{\mathcal{M}}(x)$, where $k_{\mathcal{M}} : \Delta \rightarrow Ord$

Entailment

- $\mathbf{T}(C) \sqsubseteq D$ is preferentially (rationally) entailed by K, if it is satisfied in all preferential (ranked) models $\mathcal{M}$ of K.

A minimal model semantics for RC

Preferential and rational entailment define a weak notion of inference.

Prefer the ranked models which minimize the rank of individuals: (Casini, et al., DL 2013), (Giordano, et al., DL 2013) *Minimal canonical ranked models* of the KB provide a semantic characterization *of the rational closure* of $\mathcal{ALC}$.

It is a generalization of DLs of the canonical model semantics of Rational Closure by Lehmann and Magidor (1992).

The first Rational Closure construction for DLs was developed for $\mathcal{ALC}$ by Casini and Straccia (2010).

Rational Closure ranking

RC construction assigns a rank to each defeasible inclusion and to each concept: less exceptional concepts have lower rank.

Example

$PhDSt \sqsubseteq St$ $\quad$ $Emp \sqsubseteq Adult$

$PhDSt \sqsubseteq Adult$ $\quad$ $PrimarySchoolSt \sqsubseteq Children$

——— rank 0 ———

$\mathbf{T}(St) \sqsubseteq \neg Has\_Scholarship$, $\mathbf{T}(St) \sqsubseteq Young$ $\quad rank(St) = 0$

$\mathbf{T}(Emp) \sqsubseteq \neg Young$ $\quad rank(Emp) = 0$

——— rank 1 ———

$\mathbf{T}(PhDSt) \sqsubseteq Has\_Scholarship$, $\mathbf{T}(PhDSt) \sqsubseteq Bright$

$rank(PhDSt) = 1$

$\mathbf{T}(Emp \sqcap St) \sqsubseteq Busy$ $\quad rank(Emp \sqcap St) = 1$

——— rank 2 ———

$\mathbf{T}(Emp \sqcap St \sqcap OnHolyday) \sqsubseteq \neg Busy$

$rank(Emp \sqcap St \sqcap OnH) = 2$

Advantages and drawbacks of the rational closure

- The rational closure has *good computational properties*. and *can be extended to* lightweight and expressive DLs
- Too weak: All or nothing:
  "the blocking of property inheritance problem" [Pearl,90]
  "the drowning problem" [Benferhat,Dubois,Prade,93].
- On the other hand: some conclusions are too strong [Geffner and Pearl,92]
- Exploits a unique preference relation $<$ among individuals, but $bob <_{Student} tom$ and $tom <_{Employee} bob$.

Alternative constructions to Rational Closure:

- The lexicographic closure, originally introduced by Lehmann and, for $\mathcal{ALC}$, by (Casini and Straccia, 2012),
- The relevant closure (Casini, Meyer, Moodley, Nortje, 2014)
- The logic of overriding $\mathcal{DL}^N$ (Bonatti, Faella, Petrova, Sauro, 2015)
- Skeptical closure (Pruv, 18), MP-closure (ECSQARU'19)

Multipreferences in DLs

- (Bonatti Lutz, Wolter, 2009) circumscriptive KBs also allow *abnormal instances of a class C with respect to a given aspect P* using binary abnormality predicates $Ab(P, x)$
- (Fernandez Gil, 2014) *several typicality operators* $\mathbf{T}_1, \mathbf{T}_2$,.. and preference relations $<_1, <_2, \ldots$ in $\mathcal{ALC} + \mathbf{T}_{min}$
- (Gliozzi, 2016) multiple (ranked) preferences associated to *aspects* (concepts) $<_{A_1}, <_{A_2}, \ldots$; a refinement of RC

Modular multi-concept knowledge bases

A modular multi-concept knowledge base K

is a tuple $\langle \mathcal{T}, \mathcal{D}, m_1, \ldots, m_k, \mathcal{A}, s \rangle$, where:

- $\mathcal{T}$ is an $\mathcal{ALC}$ TBox,
- $\mathcal{D}$ is a set of typicality inclusions, such that
$$m_1 \cup \ldots \cup m_k = \mathcal{D},$$
- $\mathcal{A}$ is an ABox, and
- s is a function associating each module m_i with a concept: if $s(m_i) = C_i$, then *C_i is the subject of m_i*.

Example

Let K be the knowledge base $\langle \mathcal{T}, \mathcal{D}, m_1, m_2, m_3, \mathcal{A}, s \rangle$, where $\mathcal{A} = \{St(mary), Emp(tom), PhDSt(bob)\}$,

TBox $\mathcal{T}$

$PhDSt \sqsubseteq St$ $\qquad$ $Emp \sqsubseteq Adult$

$PhDSt \sqsubseteq Adult$ $\qquad$ $PrimarySchoolSt \sqsubseteq Children$

Module m_1 with subject *Student*; $s(m_1) = St$

$\mathbf{T}(St) \sqsubseteq \neg Has\_Scholarship$ $\qquad$ $\mathbf{T}(St) \sqsubseteq Young$

$\mathbf{T}(PhDSt) \sqsubseteq Has\_Scholarship$ $\qquad$ $\mathbf{T}(PhDSt) \sqsubseteq Bright$

$\mathbf{T}(Emp \sqcap St) \sqsubseteq Busy$

$\mathbf{T}(Emp \sqcap St \sqcap OnLeave) \sqsubseteq \neg Busy$

Module m_2 with subject *Employee*; $s(m_2) = Emp$

$\mathbf{T}(Emp) \sqsubseteq \neg Young$ $\qquad$ $\mathbf{T}(Emp \sqcap St) \sqsubseteq Busy$

$\mathbf{T}(Emp \sqcap St \sqcap OnLeave) \sqsubseteq \neg Busy$

Multipreference semantics for modular KBs

We introduce a preferential relation $<_i$ for each module m_i.

We exploit the *lexicographic closure semantics* which already accounts for *specificity within the modules*.

Another option for *small modules*, one for each distinguished concept C_i, containing "only" the typicality inclusions $\mathbf{T}(C_i) \sqsubseteq D$: *user defined ranks* in *ranked $\mathcal{EL}^+_\bot$ KBs* *[ICLP2020]*.

Module m_1 with subject *St* (Student), contains:

$\mathbf{T}(St) \sqsubseteq \neg Has\_Scholarship$	*rank = 0*
$\mathbf{T}(St) \sqsubseteq Young$	*rank = 1*
$\mathbf{T}(St) \sqsubseteq \exists has\_Classes.T$	*rank = 2*

Module m_2 with subject *PhDSt* (PhD Student), contains:

$\mathbf{T}(PhDSt) \sqsubseteq Has\_Scholarship$	*rank = 0*
$\mathbf{T}(PhDSt) \sqsubseteq Bright$	*rank = 1*

In this case, treatment of *specificity among modules* is required.

Lexicographic multipreferences for modular KBs

The lexicographic closure (LC)

- Introduced by Lehmann (1995) in terms of *maxiconsistent sets* and in terms of *preferential models*;
- Casini and Straccia have defined the lexicographic closure for $\mathcal{ALC}$ (2012) based on maxiconsistent sets.

Lehmann's preferential semantics of LC

A preference relation $<$ on the set of propositional interpretations (a *modular partial order relation*):

$$w < w' \;\; \textbf{\textit{iff}} \; V(w) \prec V(w')$$

where $V(w)$ is *the set of defaults violated in w* and $\prec$ is a *seriousness ordering* among sets of defaults.
(the defaults violated in w are less serious than the defaults violated in w').

The lexicographic closure semantics for $\mathcal{ALC}$

Let $\langle \Delta, \cdot^I \rangle$ be a (finite) $\mathcal{ALC}$ model of K.

We define a preferential (ranked) model $\mathcal{N} = \langle \Delta, <, \cdot^I \rangle$ for K.

Preference relation $<$ on Δ

We define a *(modular) partial order relation* $<$ on Δ as follows:

$$x < y \ \textbf{\textit{iff}}\ V(x) \prec V(y)$$

where $V(x)$ is *the set of defaults violated in x* and $\prec$ is a *seriousness ordering* among sets of defaults.
(the defaults violated in x are less serious than the defaults violated in y).

Note: $<$ as in Lehmann's preference semantics but preferences are over Δ.

Lexicographic models for module m_i

The *projection of the knowledge base K on module m_i* as the knowledge base $K_i = \langle \mathcal{T} \cup m_i, \mathcal{A} \rangle$.

A lexicographic model of K_i

is a *preferential model* $\mathcal{N}_i = \langle \Delta, <_i, \cdot^I \rangle$ of K_i, such that:

- $\langle \Delta, \cdot^I \rangle$ is an $\mathcal{ALC}$ model of $\langle \mathcal{T}, \mathcal{A} \rangle$ and
- $<_i$ satisfies the following condition:

$$x <_i y \;\; \textbf{\textit{iff}} \; V_i(x) \prec_i V_i(y). \tag{1}$$

where $V_i(x)$ is the set of *defaults in K_i violated by x*.

A lexicographic model $\mathcal{N}_i = \langle \Delta, <_i, \cdot^I \rangle$ of $K_i = \langle \mathcal{T} \cup m_i, \mathcal{A} \rangle$ is a *ranked model* of K_i.

Multiconcept lexicographic models

A multi-concept model for K is a *multi-preference interpretation* with *a preference relation $<_i$ for each module m_i*.
Let $K = \langle \mathcal{T}, \mathcal{D}, m_1, \ldots, m_k, \mathcal{A}, s \rangle$ be a multi-concept knowledge base.

Multi-concept lexicographic models

A *multi-concept lexicographic model* $\mathcal{M} = \langle \Delta, <_1, \ldots, <_k, \cdot^I \rangle$ of K is a multi-concept interpretation for K, such that, for all $i = 1, \ldots, k$, $\mathcal{N}_i = \langle \Delta, <_i, \cdot^I \rangle$ is a lexicographic model of $K_i = \langle \mathcal{T} \cup m_i, \mathcal{A} \rangle$.

We will consider *canonical* multi-concept lexicographic models of K, where the domain Δ contains, roughly speaking, *as many domain elements as consistent with K*.

Reasoning in a multi-concept knowledge base

In a multi-concept lexicographic model $\mathcal{M} = \langle \Delta, <_1, \ldots, <_k, \cdot^I \rangle$ of K, preference relation $<_i$ can be used to answer *queries over module m_i* (i.e. queries with subject C_i).

The query *"Are all typical Phd students young?"* can be evaluated in module m_2 with subject *Student* by verifying that in all canonical multiconcept models of K,

$$min_{<_2}(PhD\_Student^I) \subseteq Young^I.$$

The answer would be positive, as the property of students of being normally young is inherited by PhD Student.

But what about the query:

"Are typical employed students young?"

Employed students are concerned with both subjects *Student* and *Employee*, i.e., with both modules m_1 and m_2, and with both perferences $<_1$ and $<_2$.

Combining preferences

We need to *combine the preference relations* $<_i$ into a single one $<$.

Let $\leq_i$ be defined as follows: $x \leq_i y$ iff $y \not<_i x$.
As $<_i$ is a modular partial order, $\leq_i$ is a total preorder.

Given a canonical multi-concept lexicographic model $\mathcal{M} = \langle \Delta, <_1, \ldots, <_k, \cdot^I \rangle$ of K, we define a *global preference relation* $<$ on Δ by Pareto combination:

$$x < y \quad \textit{iff} \quad (i) \text{ for some } i = 1, \ldots, k,\ x <_i y \text{ and} \qquad (*)$$
$$(ii) \text{ for all } j = 1, \ldots, k, x \leq_j y,$$

$<$ *is a partial order relation* (but, in general, modularity does not hold for $<$).

$\mathcal{M}^{\mathbf{P}} = \langle \Delta, <, \cdot^I \rangle$ is a *combined lexicographic model of K*.

Combined lexicographic models of K and entailment

A combined lexicographic model $\mathcal{M}^{\mathbf{P}} = \langle \Delta, <, \cdot^I \rangle$ of K:

- is a preferential interpretation;
- *satisfies strict inclusions and assertions* in K;
- but is *not* required to satisfy *all typicality inclusions* $\mathbf{T}(C) \sqsubseteq D$ in K.

Example

$s(m_1) = Student$

$\mathbf{T}(Student) \sqsubseteq YoungPerson$ $\quad$ $\mathbf{T}(Student) \sqsubseteq Quiet$

$s(m_2) = YoungPerson$:

$\mathbf{T}(YoungPerson) \sqsubseteq Student$ $\quad$ $\mathbf{T}(YoungPerson) \sqsubseteq \neg Quiet$

If Bob and John be young persons and also students, and $bob <_1 john$ and $john <_2 bob$ (they are incomparable wrt $<$)

It may be that:

$$bob, john \in min_<(Student^I) \not\subseteq min_{<_1}(Student^I)$$

(note that this KB has *no KLM style preferential model*)

Multi-concept lexicographic entailment

To require that all typicality inclusions in K are satisfied in $\mathcal{M}^{\mathbf{P}}$, the notion of m_l^c-model of K can be strengthened by defining **T**-*compliant* m_l^c*-models* of K, i.e., m_l^c-models of K satisfying all typicality inclusions in K.

The notions of m_l^c*-entailment* and **T**-*compliant* m_l^c*-entailment* can be defined in the obvious way.

- *satisfy the KLM postulates of preferential consequence relations*, which can be reformulated for a typicality logic, considering that typicality inclusions $\mathbf{T}(C) \sqsubseteq D$ stand for conditionals $C \mid\!\sim D$.
- *avoid the drowning problem* (as lexicographic closure does)
- *avoid unwanted inferences* of RC and LC (if proper modularization)

Avoiding too strong conclusions of RC

- The *refinements of RC* (including MP-closure) avoid the "inheritance blocking" but *allow for too strong conclusions* (Geffner and Pearl, 92).
- To deal correctly with ambiguity preservation *Conditional Entailment* (Geffner and Pearl, 92) and *system LCD* (Benferhat, Saffiotti and Smets, 2000) abandon Rational Monotonicity (RM). *System JLZ* (Weydert, 2003) exploits a canonical ranking construction and verifies (RM).
- *Combining multiple preferences* is a simple approach which may avoid too strong conclusions (depending on the definition of specificity $\prec$) as well as inheritance blocking. This is the case for the *concept-wise multipreference semantics*.

Example: ambiguity preservation

A reformulation of an example by Geffner and Pearl (AIJ'92).

Consider module m_1 with subject *Citizen*:

$\mathbf{T}(\mathit{Italian}) \sqsubseteq \mathit{DriveFast}$

$\mathbf{T}(\mathit{Italian}) \sqsubseteq \mathit{HomeOwner}$

and module m_2 with subject *Student*:

$\mathbf{T}(\mathit{PhDStudent}) \sqsubseteq \neg \mathit{HomeOwner}$

$\mathbf{T}(\mathit{PhDStudent}) \sqsubseteq \mathit{Has\_Scholarship}$

In RC and LC we would neither conclude that Italian PhD students are home owners nor that they are not home owners. If the inclusion

$\mathbf{T}(\mathit{Student}) \sqsubseteq \neg \mathit{Has\_Scholarship}$

is added, being $\mathit{PhDStudent} \sqsubseteq \mathit{Student}$, RC and LC conclude:

$\mathbf{T}(\mathit{Italian} \sqcap \mathit{PhDStudent}) \sqsubseteq \neg \mathit{HomeOwner}$

("the conflict is resolved anomalously").

Separating typicality inclusions in the two modules m_1 and m_2 avoids this conclusion.

A hierarchy of modules

We may have introduce different modules

$$m_1, m_2, m_3, m_4$$

with subject (resp.)

Student, *Employee*, *PhDStudent*, $Student \sqcap Employee$.

A *specificity relation* $\succ$ among concepts (modules) has to be taken into account:

$PhDStudent \succ Student$

(*PhDStudent* is more specific than *Student*)

$Student \sqcap Employee \succ Student$

$Student \sqcap Employee \succ Employee$

Specificity is needed for defining the global preference $<$.

We can adopt the *modified Pareto condition* from [TPLP2020].

$$x < y \text{ iff } \quad (i)\ x <_i y, \text{ for some } C_i \in \mathcal{C}, \text{ and}$$
$$(ii)\ \text{ for all } C_j \in \mathcal{C},\ x \leq_j y \text{ or } \exists C_h (C_h \succ C_j \text{ and } x <_h y)$$

The preference relation $<_3$ for PhD students overrides the preference relation $<_1$ for Students.

Conclusions

- *Combining multiple preferences* is a simple approach which may avoid too strong conclusions of RC as well as inheritance blocking. This is the case for the *concept-wise multipreference semantics*.
- The *approach to preference combination* has different instantiations depending on: the *granularity of aspects* (e.g., concepts or modules), the definition of *preferences* $<_{A_i}$ and the notion *specificity* [NMR2020].
- An algebraic framework for preference combination has been proposed by Bozzato et al. [TPLP2021].
- *Plausibility* of a *concept-wise semantics* has been supported [Cilc 2020] by showing that it can be used to define a *logical interpretation* of *self-organising maps (SOMs)*, psychologically and biologically plausible neural network models (Kohonen'01).

Related work

- Brewka's *Preferred subtheories* (1998) and framework of *Basic Preference Descriptions* (2004)
- *Preference fusion* to define *system ARS* (Kern-Isberner and Ritterskamp, 2010) a refinement System Z;
- *Syntax Splitting* of conditional KBs for c-representations (Kern-Isberner, Beierle, Brewka, 2020);
- Gil (2014) has defined a multipreference formulation of the typicality DL $\mathcal{ALC} + \mathbf{T}_{min}$ to avoid inheritance blocking;
- Gliozzi's multipreference semantics (2016) to define a *refinement of rational closure*.
- Britz and Varzinczak (2018, 2019) associate *preference relations to roles*, to define *defeasible role quantifiers* and *defeasible role inclusions*;
- Delgrande and Rantsoudis (NMR 2020) have proposed a *multi-preferential approach* for representing defaults in FOL

Related work

- In the CKR (*Contextualized Knowledge Repositories*) framework, by Bozzato, Eiter and Serafini (2014,2018), defeasible axioms are allowed and exceptions can be handled by overriding. Extended to *general contextual hierarchies* (2019). *ASP based reasoning* for $\mathcal{SROIQ}$-RL.
- In [ICLP 2020] the approach is applied to *ranked* $\mathcal{EL}^{+}_{\bot}$ *knowledge bases*, where each module corresponds to a concept C_i and only contains defaults $\mathbf{T}(C_i) \sqsubseteq D$ (with their ranks). ASP (and *asprin*) used for defeasible reasoning in ranked $\mathcal{EL}^{+}_{\bot}$ KBs.
- An *algebraic framework for preference combination* has been proposed by Bozzato, Eiter and Kiesel [TPLP2021].
- *Weighted defeasible DL knowledge bases* for modeling *Multilayer Perceptrons* under a fuzzy multipreference semantics [JELIA 2021]. For the two-valued preferential semantics ASP based reasoning in ICLP2021.

Thank you!!!!!