An Agent-based Model on scale-free networks for Personal Finance Decisions

L. Mastroeni[1] M. Naldi[2] **Pierluigi Vellucci**[1]

[1]Department of Economics, University of Roma TRE
[2]Dept. of Civil Engineering and Computer Science, University of Rome Tor Vergata

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Outline

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Personal finance decisions

Personal finance decisions

- variety of factors:
  - recommendations from individual stock brokers
  - perceived ethics of a firm
- several agents

Personal finance decisions

Society

Categories: banks, financial advisors, investors

Society: "scale-free" networks

- Investors resort to FAs to improve their investment process
- FAs are paid by banks
- The aim of banks is to steer the investors towards a particular investment decision (collaboration of FAs)
- The interaction is not of the any-to-any kind (e.g., investors connect with advisors but not with banks)
- Some agents have a large number of connections to other individuals, whereas most of them just have a handful (e.g., an advisor may have many customers, but customers usually have only one advisor)
- Agent-Based Model (ABM)

Contributions of the paper

We extend the bounded confidence model of continuous opinion formation introduced in

- M. Steinbacher and M. Steinbacher, *Opinion formation with imperfect agents as an evolutionary process*, Computational Economics

by introducing Gaussian updating functions

- G. Deffuant, *Comparing extremism propagation patterns in continuous opinion models*, Journal of Artificial Societies and Social Simulation

Why Gaussian updating function?

According to classical bounded confidence models, the agents interact with each other only when their opinions are close enough. But in many real world situations, the strength of this interaction usually depends on the distance between opinions (the lower the distance, the higher the strength).

Contributions of the paper

Several categories of agents: honest agents, regular agents, insincere agents, stubborn agents and skilled (or unskilled) agents.

- Honest agents truthfully report their opinion while insincere agents state an opinion that may be different from their internal belief
- Regular agents are characterized by a common propensity to listen
- Stubborn agents evaluate the counterpart's opinion but do not change their opinions at all
- Skilled and unskilled agents:
    - L. Pareschi, P. Vellucci, and M. Zanella, *Kinetic models of collective decision-making in the presence of equality bias*

Contributions of the paper

- All these subsets have empty intersections (partition of the set of all agents)
- Focus only on scale free networks.

Outline

Description of the model

- Graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$
- Finite set of agents $i \in \mathcal{V} = \{1, 2, \ldots, n\}$:
  - nodes on a network
  - connected to each other with a finite set of links $\mathcal{E}$
- Bilateral interaction, i.e. $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$
- Each agent is characterized by the following triple:
  - a couple of opinions, threshold level and a set of connections
  - $i = \left\{ \left(x_i(t), x_i^R(t) \right), \epsilon_i, \mathcal{N}_i \right\}$
- All the opinions fall in the range $[0, 1]$ and are related to the decisions to buy a security rather than a different security or other financial instruments.

Description of the model

- Time t, agent i selects a random counterpart j from his neighborhood $\mathcal{N}_i = \{j \in \mathcal{V} | (j,i) \in \mathcal{E}\}$
- $x_i^R(t)$ is the opinion that agent i reports to the selected counterpart
- Threshold levels are assigned to each agent at $t = 0$, with $\epsilon_i \in [0,1]$.

Insincere vs honest

- $x_i^R(t) \neq x_i(t)$ in the case of *insincere* agents;
- $x_i^R(t) = x_i(t)$ in the case of *honest* agents.

Description of the model

Updating function

Agents adjust their opinion upon the principle of bounded confidence:

$$\Delta x_i(t) = x_i(t+1) - x_i(t)$$
$$\Delta x_j(t) = x_j(t+1) - x_j(t)$$

and

$$\Delta x_i(t) = \mu\, e^{-\left(x_i(t)-x_j^R(t)\right)^2} \chi_{(-\epsilon_i,\epsilon_i)}\left(d_{i,j}(t)\right)\left(x_j^R(t) - x_i(t)\right)$$
$$\Delta x_j(t) = \mu\, e^{-\left(x_i(t)-x_j^R(t)\right)^2} \chi_{(-\epsilon_i,\epsilon_i)}\left(d_{i,j}(t)\right)\left(x_i^R(t) - x_j(t)\right)$$

Description of the model

Updating function

- $\mu \in [0,1]$ is the *adoption rate*, representing the proportion of counterpart's opinion an agent integrates into his prior
- $d_{i,j}(t) = x_i(t) - x_j^R(t)$
- $\chi_{(-\epsilon_i,\epsilon_i)}(x)$ is the characteristic function of the interval $(-\epsilon_i, \epsilon_i)$

Stubborn vs non-stubborn

- A *stubborn* agent i has parameter values of $\epsilon_i = 0 \vee \mu = 0$
- A *non-stubborn* agent i has parameter values of $\epsilon_i > 0 \wedge \mu > 0$

Description of the model

Regular agents

Threshold levels are equal across the population of regular agents, i.e. $\epsilon_1 = \epsilon_2 = \cdots = \epsilon_n$.

Skilled and unskilled agents

The influence of competence in the evolution of decisions in multi-agent systems.

Opinion vector

$$\mathbf{x}(t) = (x_1(t), \ldots, x_n(t))^T .$$

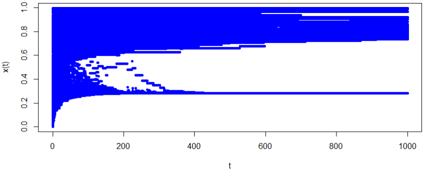
Outline

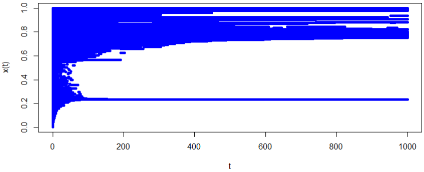
Opinion Formation with Regular, Honest Agents

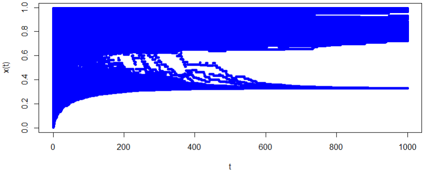
- Simplest case (only regular agents are present)
- Initial opinion vector $\mathbf{x}(0)$: standard uniform distribution with $[0, 1]$ support
- All agents have the same threshold level $\epsilon_i = \epsilon$, the same adoption rate and $x_i^R(t) = x_i(t)$ for every i (i.e., all agents are honest).

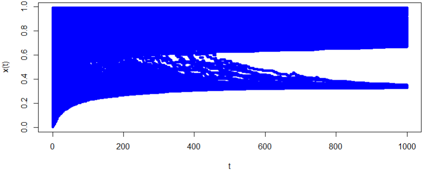
Opinion Formation with Regular, Honest Agents

Here $\epsilon = 0.3$ and $n = 500$ agents.







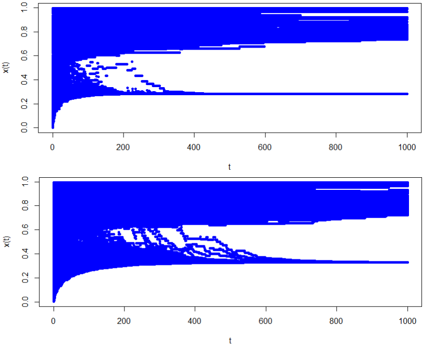


$\mu = 0.3$ (top plot) and $\mu = 0.1$ (bottom plot).

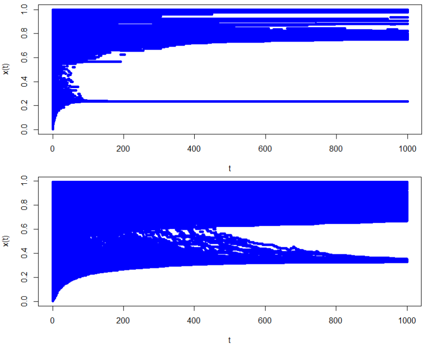
$\mu = 0.5$ (top plot) and $\mu = 0.05$ (bottom plot).

Opinion Formation with Regular, Honest Agents

I) Agents that have an initial starting opinion below a certain threshold (between 0.5 and 0.6, top plots) are rapidly drawn to a low central consensus; μ speeds up the convergence to the low central consensus



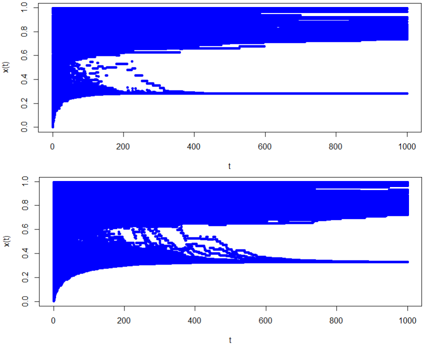
$\mu = 0.3$ (top plot) and $\mu = 0.1$ (bottom plot).



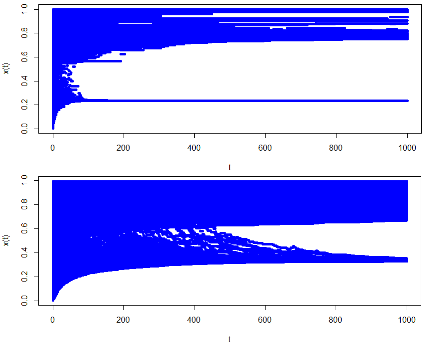
$\mu = 0.5$ (top plot) and $\mu = 0.05$ (bottom plot).

Opinion Formation with Regular, Honest Agents

II) Agents outside the threshold settle down to a larger number of extreme opinions (isolated from the low central consensus)



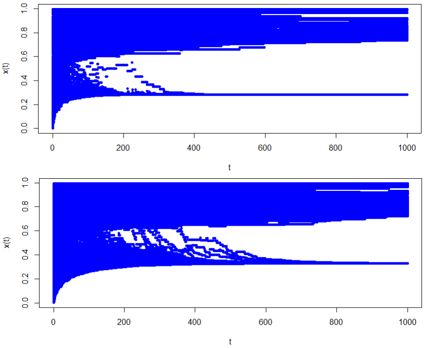
$\mu = 0.3$ (top plot) and $\mu = 0.1$ (bottom plot).



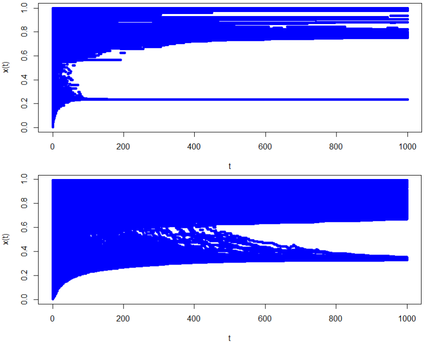
$\mu = 0.5$ (top plot) and $\mu = 0.05$ (bottom plot).

Opinion Formation with Regular, Honest Agents

III) As μ increases, high extreme opinions become more and more distinguishable (μ represents the proportion of counterpart's opinion an agent integrates into its own).



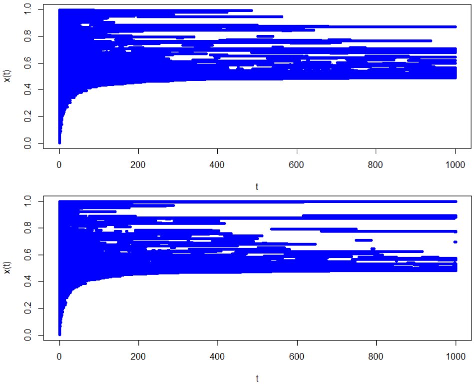
$\mu = 0.3$ (top plot) and $\mu = 0.1$ (bottom plot).



$\mu = 0.5$ (top plot) and $\mu = 0.05$ (bottom plot).

Opinion Formation with Regular, Honest Agents

The following figure underlines this effect by fixing μ and considering increasing, high values of threshold ϵ. As ϵ rises, high extreme opinions emerge. Parameters: $\mu = 0.3$ and $n = 500$ agents; $\epsilon = 0.7$ (top plot) and $\epsilon = 0.9$ (bottom plot).

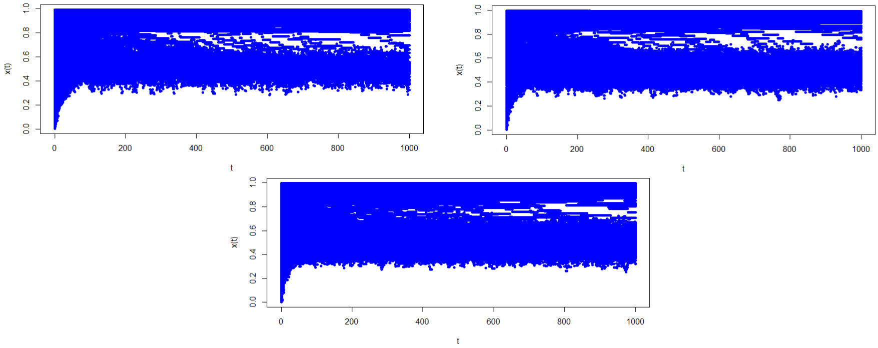


Opinion Formation with Insincere Agents

- Stated opinion $x_i^R(t)$: standard uniform distribution with $[0,1]$ support
- Society $\mathcal{V}$ can be subdivided into two subsets, $\mathcal{H}$ (honest agents) and $\mathcal{I}$ (insincere agents), such that $\mathcal{V} = \mathcal{H} \cup \mathcal{I}$ and $\mathcal{H} \cap \mathcal{I} = \emptyset$
- All the agents are regular, i.e. $\epsilon_1 = \epsilon_2 = \cdots = \epsilon_n$
- Society $\mathcal{V}$ of $n = 500$ agents (m of them are insincere)

Opinion Formation with Insincere Agents

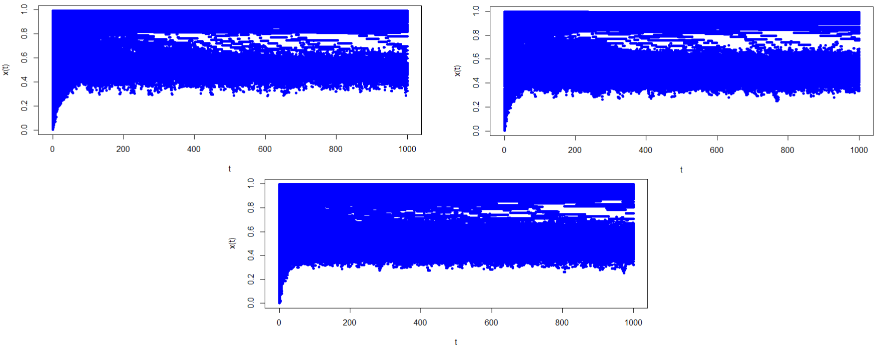
In the following figure we examined the impact of the total number of insincere agents, m, on the opinion dynamics.



Parameters: $m = 30$ (TL plot); $m = 150$ (TR plot); $m = 250$ (bottom plot). Here $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

Opinion Formation with Insincere Agents

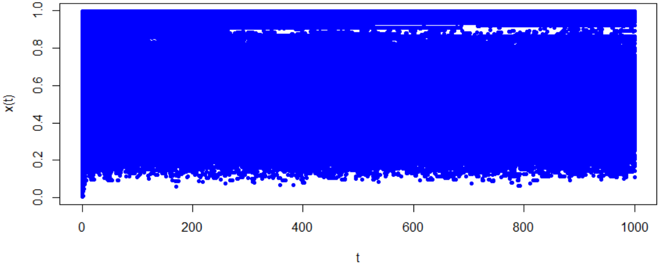
Low central consensus disappeared and a disordered regime emerged (opinions are in a constant state of change around a central opinion)



Parameters: $m = 30$ (TL plot); $m = 150$ (TR plot); $m = 250$ (bottom plot). Here $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

Opinion Formation with Insincere Agents

Increasing the willingness to listen (by increasing μ) does not improve the picture. In the presence of insincere agents, a greater proportion of counterpart's opinion that an agent is willing to accept leads to a more pronounced disordered regime.



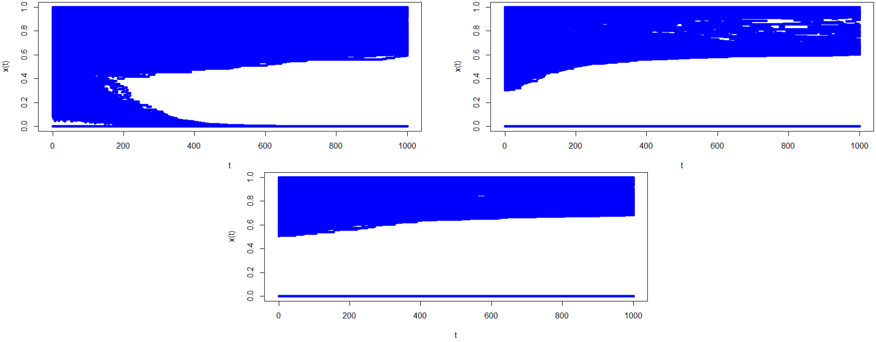
Parameters: $\mu = 0.5$, $\epsilon = 0.3$, $m = 250$ and $n = 500$.

Opinion Formation with Regular and Stubborn Agents

- $\mathcal{V}$ (entire population of agents), $\mathcal{R}$ (regular agents) and $\mathcal{S}$ (stubborn agents), such that $\mathcal{V} = \mathcal{S} \cup \mathcal{R}$ and $\mathcal{S} \cap \mathcal{R} = \varnothing$
- Stubborn agents can be described as individuals that are biased towards their initial opinions. They have the ability to exert their influence onto others but cannot be influenced by the rest of society
- Society $\mathcal{V}$ of $n = 500$ agents (m of them are stubborn)
- Stubborn agents are assigned same initial opinion $x^S = \alpha \in [0, 1]$
- Stubborn and regular agents interact

Opinion Formation with Regular and Stubborn Agents

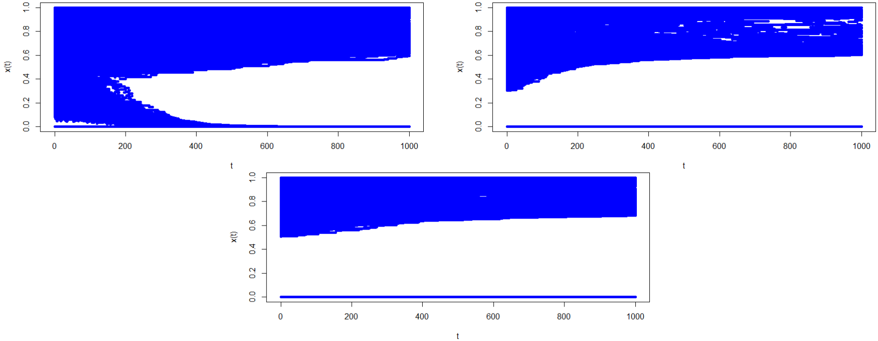
Impact of the total number of stubborn agents, m, on the opinion dynamics for $\alpha = 0$



Parameters: $m = 30$ (top-left plot); $m = 150$ (top-right plot); $m = 250$ (bottom plot); $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

Opinion Formation with Regular and Stubborn Agents

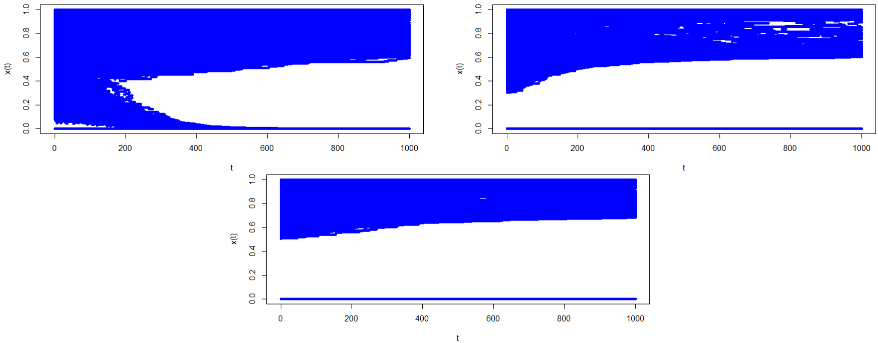
I) Presence of the $x_i = 0$ extreme opinion of the stubborn agents



Parameters: $m = 30$ (top-left plot); $m = 150$ (top-right plot); $m = 250$ (bottom plot); $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

Opinion Formation with Regular and Stubborn Agents

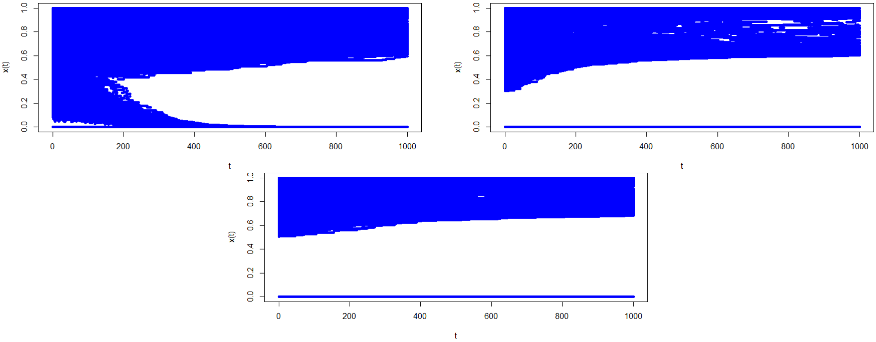
II) Low central consensus is vanished



Parameters: $m = 30$ (top-left plot); $m = 150$ (top-right plot); $m = 250$ (bottom plot); $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

Opinion Formation with Regular and Stubborn Agents

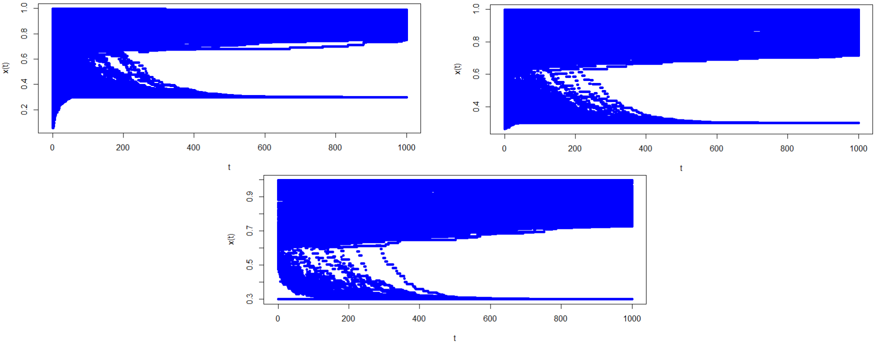
III) m not too big ($m = 30$): low central consensus deviates toward the position of stubborn agents (it disappears when the number of stubborn agents increases)



Parameters: $m = 30$ (top-left plot); $m = 150$ (top-right plot); $m = 250$ (bottom plot); $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

Opinion Formation with Regular and Stubborn Agents

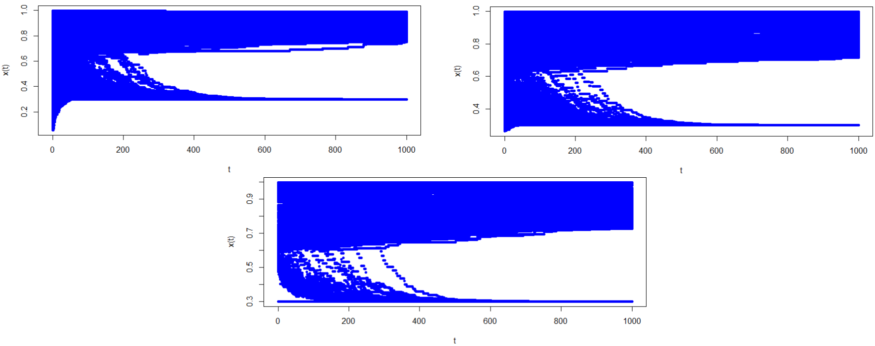
Impact of α on the time evolution of opinion dynamics, by considering $\alpha = 0.3$:



Parameters: $m = 30$ (top-left plot); $m = 150$ (top-right plot); $m = 250$ (bottom plot); $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

Opinion Formation with Regular and Stubborn Agents

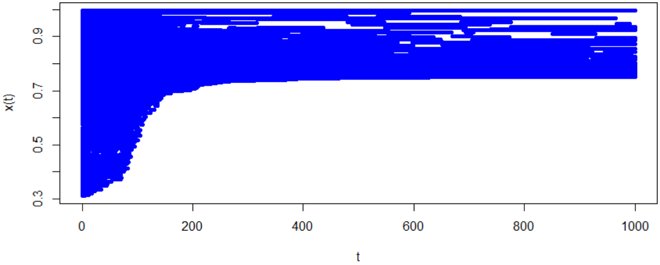
Consensus on the position of the stubborn agents persists with increasing number of stubborn agents. A consensus can be stimulated by stubborn agents, but it resists to their proliferation if $\alpha \neq 0$ (sufficiently).



Parameters: $m = 30$ (top-left plot); $m = 150$ (top-right plot); $m = 250$ (bottom plot); $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

Opinion Formation with Regular and Stubborn Agents

If α is sufficiently near to 1, the consensus cannot be reached:



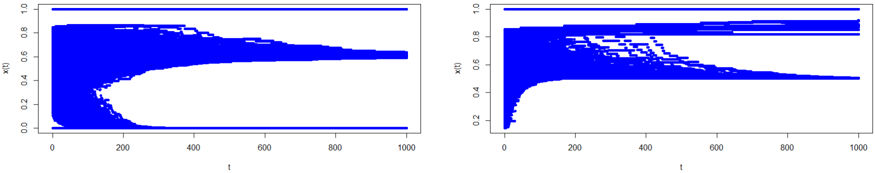
Number of stubborn agents: $m = 150$. Other parameters: $\mu = 0.1$, $\epsilon = 0.3$ and $n = 500$ agents.

Opinion Formation with Two Groups of Stubborn Agents

- Introduce a second group of stubborn agents
- Society $\mathcal{V}$ of $n = 1000$ agents ($2m$ of them are stubborn)
- Two classes of stubborn agents: initial opinions $x^{S_1} = \alpha \in [0, 1]$ and $x^{S_2} = \beta \in [0, 1]$, with $\alpha \neq \beta$.
- The groups of stubborn agents are equally sized, consisting of $m = 150$ stubborn agents each

Opinion Formation with Two Groups of Stubborn Agents

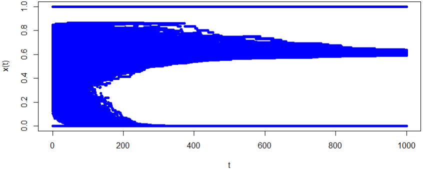
I) When $\alpha = 0$ the extremism prevails and the society ends in a complete polarisation of the opinion space

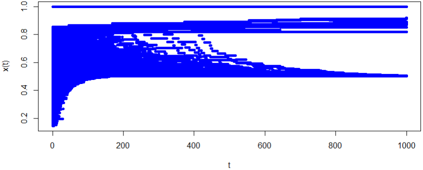


Parameters: $\beta = 1$, $\alpha = 0$ (left plot), $\alpha = 0.5$ (right plot); $\mu = 0.1$, $\epsilon = 0.3$.

Opinion Formation with Two Groups of Stubborn Agents

II) Isolated position of S_2 class of stubborn agents with $x^{S_2} = 1$

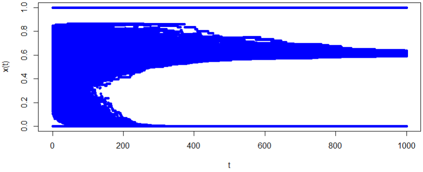


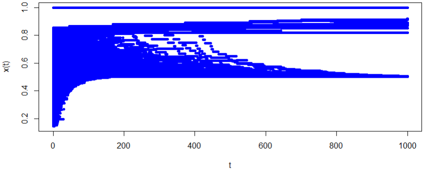


Parameters: $\beta = 1$, $\alpha = 0$ (left plot), $\alpha = 0.5$ (right plot); $\mu = 0.1$, $\epsilon = 0.3$.

Opinion Formation with Two Groups of Stubborn Agents

III) When α reaches 0.5 the majority of regular agents concentrate in the center, forming a large single opinion class. If we denote this class by $\mathcal{C}$, we have that for $T \to +\infty$, $x_i(T) \to 0.5 \ \forall i \in \mathcal{C}$

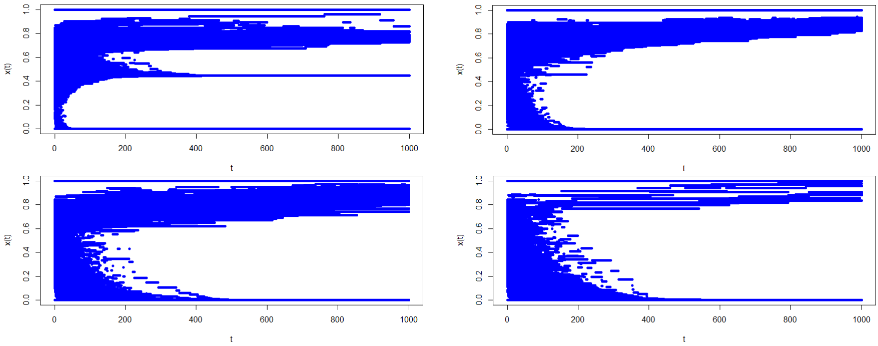




Parameters: $\beta = 1$, $\alpha = 0$ (left plot), $\alpha = 0.5$ (right plot); $\mu = 0.1$, $\epsilon = 0.3$.

Opinion Formation with Two Groups of Stubborn Agents

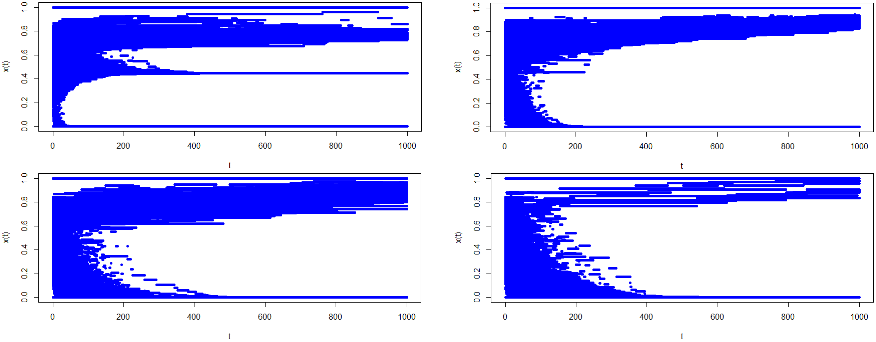
Impact of ϵ on the opinion formation process with regular agents and two groups of stubborn agents:



Parameters: $\alpha = 0$ and $\beta = 1$; $\epsilon = 0.2$ (TL), $\epsilon = 0.45$ (TR), $\epsilon = 0.55$ (BL) and $\epsilon = 0.7$ (BR); $\mu = 0.3$.

Opinion Formation with Two Groups of Stubborn Agents

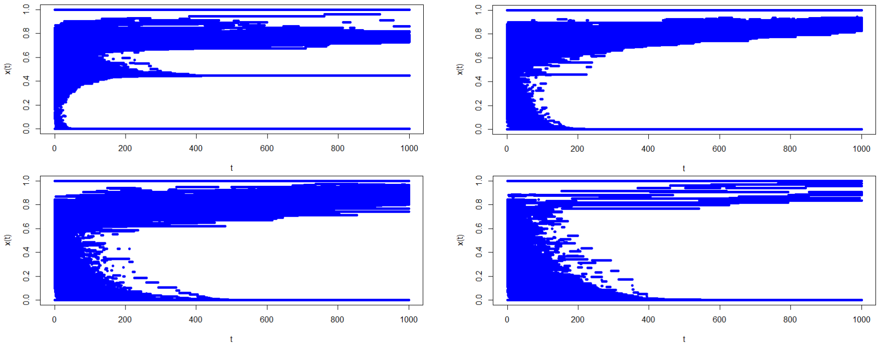
I) When $\epsilon < 0.3$ the agents whose opinion is approximately in the range $[0, 0.6]$ move towards either a central consensus or the position of S_1.



Parameters: $\alpha = 0$ and $\beta = 1$; $\epsilon = 0.2$ (TL), $\epsilon = 0.45$ (TR), $\epsilon = 0.55$ (BL) and $\epsilon = 0.7$ (BR); $\mu = 0.3$.

Opinion Formation with Two Groups of Stubborn Agents

II) As ϵ rises, the central consensus vanishes and only $x^{S_1} = 0$ remains in addition to the high extreme opinions.



Parameters: $\alpha = 0$ and $\beta = 1$; $\epsilon = 0.2$ (TL), $\epsilon = 0.45$ (TR), $\epsilon = 0.55$ (BL) and $\epsilon = 0.7$ (BR); $\mu = 0.3$.

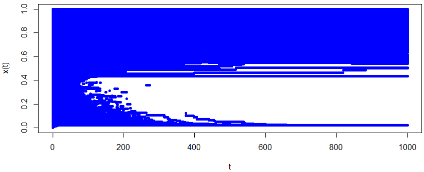
Outline

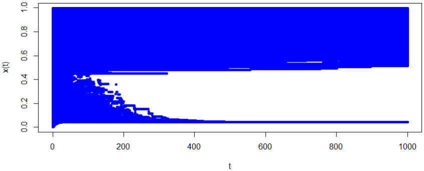
Skilled Regular, Honest Agents

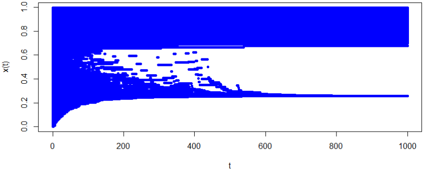
- More well-educated and competent people are also those best disposed to dialogue
- An agent with an attitude to listen other people is characterized by a high competence, while an individual unwilling to listen and dialogue is usually marked by a lower level of the described trait
- $\mathbf{y} = (y_1, \dots, y_n)^T$: the competence vector, constant in time (standard uniform distribution with $[0, 1]$ support for the first m agents, $[10, 15]$ support for the remaining ones)
- The initial opinion vector $\mathbf{x}(0)$ s.t. $x_1(0) < x_2(0) < \dots < x_n(0)$
- The threshold of Gaussian bounded confidence model depends on the degree of competence: $\epsilon_{i,j} = \frac{\epsilon}{1+e^{c(y_j - y_i)}}$ with $c \gg 1$

Skilled Regular, Honest Agents

I) The system evolves toward two clusters, characterizing two subpopulations with different decisions driven by the most competent agents (upper part of the plot) and the less skilled ones (lower part)



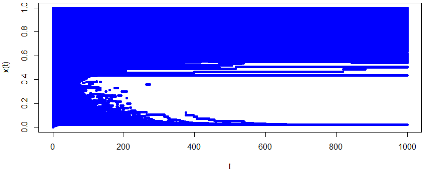


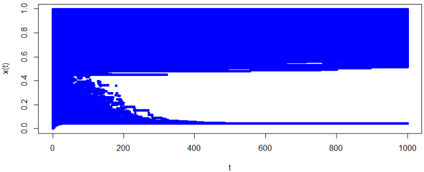


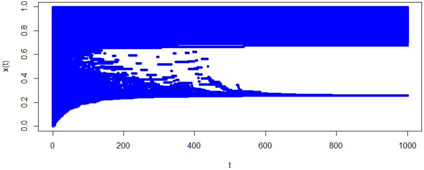
Parameters: $m = 50$ (TL plot); $m = 100$ (TR plot); $m = 500$ (bottom plot); $\mu = 0.2$, $\epsilon = 0.4$ and $n = 1000$ agents.

Skilled Regular, Honest Agents

II)We can spot the presence of a region in which regular skilled agents continuously change their opinions, in the upper part of the plot, and the presence of a lower consensus for the unskilled people







Parameters: $m = 50$ (TL plot); $m = 100$ (TR plot); $m = 500$ (bottom plot); $\mu = 0.2$, $\epsilon = 0.4$ and $n = 1000$ agents.

Outline

Conclusions and Future Perspectives

- Regular, honest agents:
  - agents with an initial starting opinion that is below a certain threshold are rapidly drawn to a low central consensus
  - μ speeds up the convergence to the low central consensus
  - as ϵ rises, high extreme opinions emerge
- Insincere agents:
  - the low central consensus disappeared
  - a disordered regime in which opinions are in a constant state of change around a central opinion, emerged by varying the number of insincere agents
  - a greater proportion of counterpart's opinion, that an agent integrates into his prior, leads to a more pronounced disordered regime

Conclusions and Future Perspectives

- Non regular agents, in presence of stubborn agents,
  - if the number of these agents is not too big, the low central consensus deviates toward the position of stubborn agents but it disappears with the increase of this number
- When another population of stubborn agents is added, the extremism prevails and the society ends in a complete polarisation of the opinion space.
- The system of skilled and unskilled agents evolves toward two clusters; regular skilled agents continuously change their opinions, while the presence of a lower consensus is due to the unskilled people.

Conclusions and Future Perspectives

- Future work: collection of large amount of user interaction information from online social networks and the analysis of the dynamic sentiments of users to investigate realistic opinion evolution, as proposed in
  - F. Xiong and Y. Liu. *Opinion formation on social media: an empirical approach*

Thanks for your attention! Any questions?

Scale-free Network

- A discrete time step model and in each time step a single vertex is added
  - Start with a single vertex and no edges in the first time step
  - Add one vertex in each time step and the new vertex initiates some edges to old vertices
  - The probability that an old vertex is chosen is given by $P_i = k_i^{\alpha} + a$ where $\alpha = a = 1$, k_i is the number of adjacent edges of i which were not initiated by i itself.